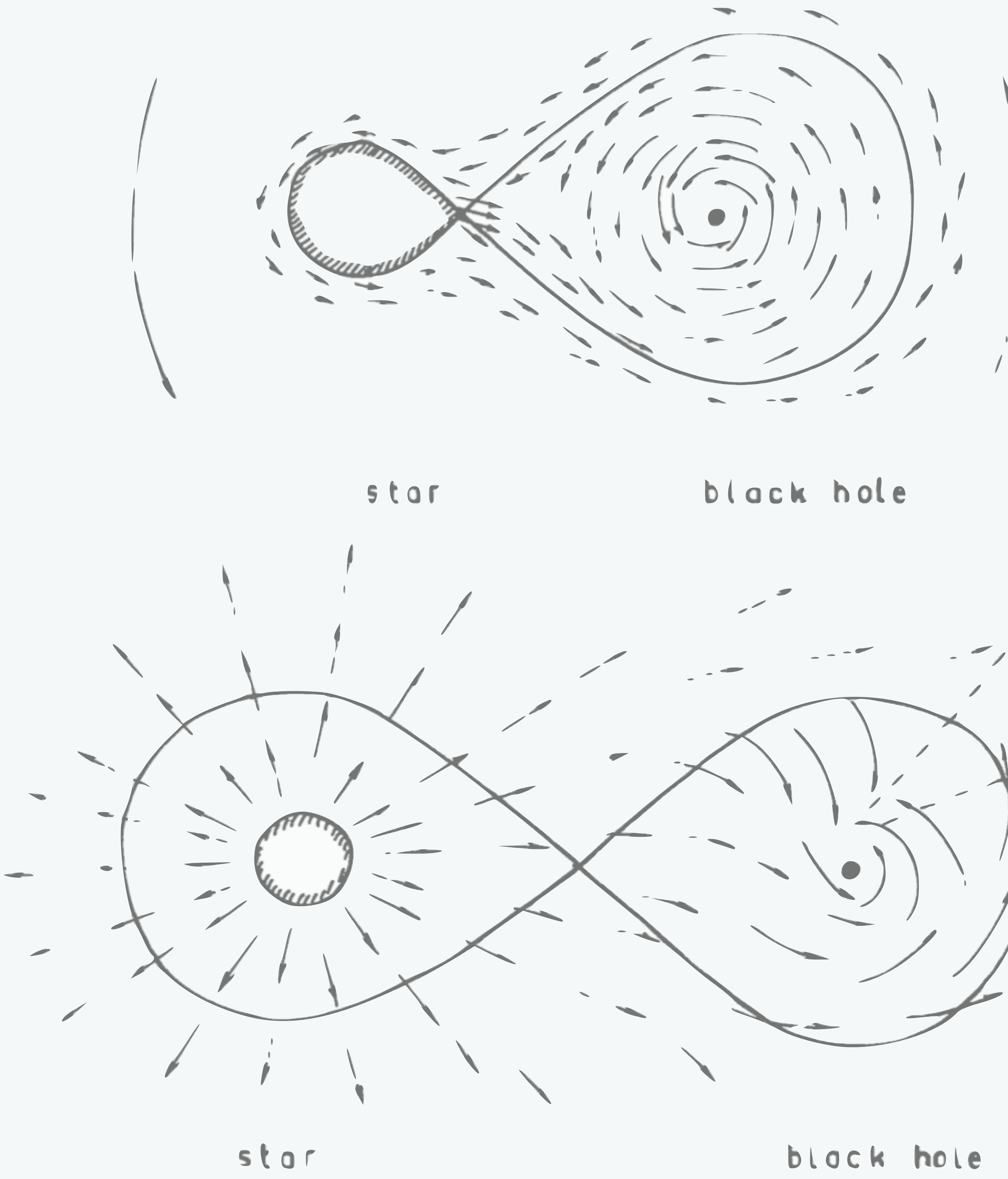
Shakura-Sunyaev disk



Yoonsoo Kim

Jul 11 2022 @ ARC journal club, Caltech

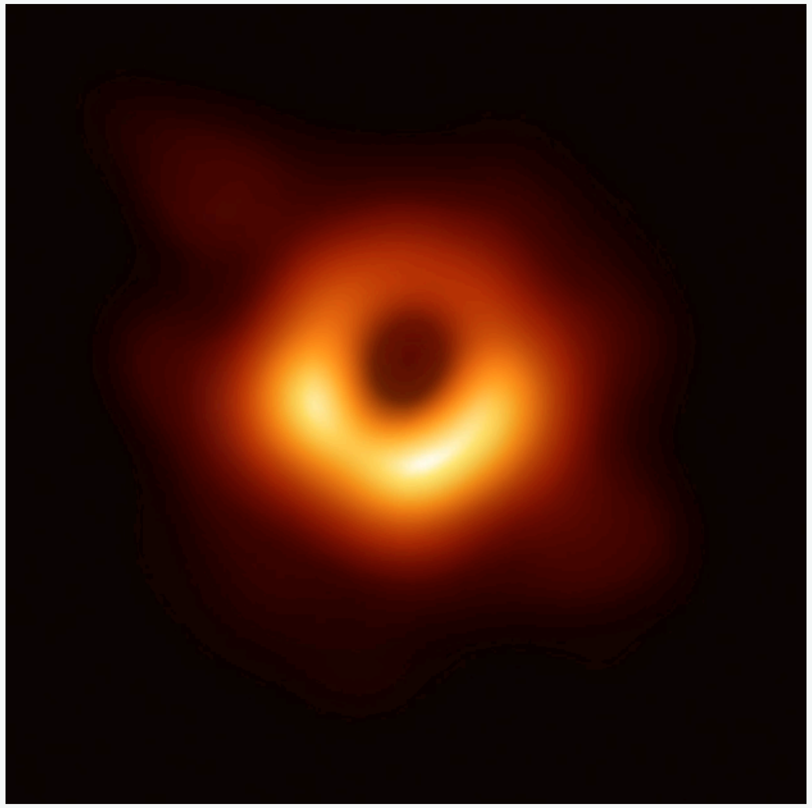
Accretion disk

Accretion disk

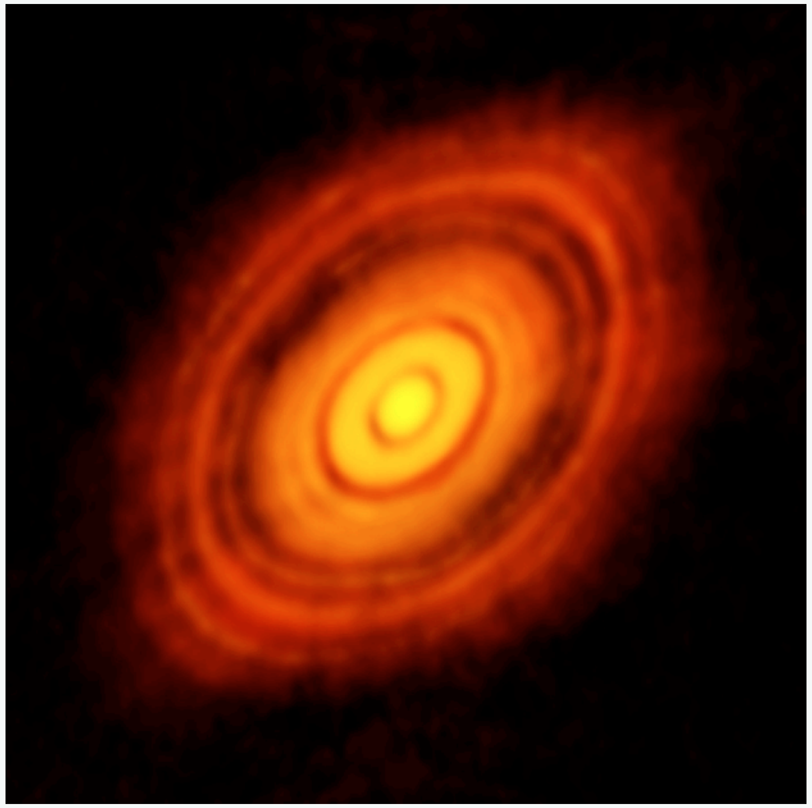
- X-ray binaries
- Active galactic nuclei (AGNs)
- Protoplanetary disks
- Tidal disruption events (TDEs)
- Post-merger remnant of compact binary merger (BH-NS or NS-NS)
- ….

Accretion disk

- X-ray binaries
- Active galactic nuclei (AGNs)
- Protoplanetary disks
- Tidal disruption events (TDEs)
- Post-merger remnant of compact binary merger (BH-NS or NS-NS)
- ….



https://en.wikipedia.org/wiki/Accretion_disk



https://en.wikipedia.org/wiki/Protoplanetary_disk

Accretion disk

Gas in the disk follows (almost) Keplerian orbit

Check :

Accretion disk

Gas in the disk follows (almost) Keplerian orbit

Check :

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{\nabla P}{\rho} - \nabla \Phi_G$$

Accretion disk

Gas in the disk follows (almost) Keplerian orbit

Check :

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{\nabla P}{\rho} - \nabla \Phi_G \qquad \frac{\nabla P}{\rho} \sim \frac{(nk_B T)/r}{nm_p}, \quad \nabla \Phi_G \sim \frac{GM}{r^2}$$

Accretion disk

Gas in the disk follows (almost) Keplerian orbit

Check :

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{\nabla P}{\rho} - \nabla \Phi_G \qquad \frac{\nabla P}{\rho} \sim \frac{(nk_BT)/r}{nm_p}, \quad \nabla \Phi_G \sim \frac{GM}{r^2}$$

gravitational force / pressure gradient $\sim \dfrac{GMm_p/r}{k_BT}$

Accretion disk

Gas in the disk follows (almost) Keplerian orbit

Check :

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{\nabla P}{\rho} - \nabla \Phi_G \qquad \frac{\nabla P}{\rho} \sim \frac{(nk_B T)/r}{nm_p}, \quad \nabla \Phi_G \sim \frac{GM}{r^2}$$

gravitational force / pressure gradient $\sim \dfrac{GMm_p/r}{k_B T}$ ~ *500*

(1 M_sun BH accretor, T = 10keV, r ~ 100 r_g)

Accretion disk

Gas in the disk follows (almost) Keplerian orbit

Check :

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{\nabla P}{\rho} - \nabla \Phi_G \qquad \frac{\nabla P}{\rho} \sim \frac{(nk_BT)/r}{nm_p}, \quad \nabla \Phi_G \sim \frac{GM}{r^2}$$

gravitational force / pressure gradient $\sim \frac{GMm_p/r}{k_BT}$ *~ 500*
(1 M_sun BH accretor, T = 10keV, r ~ 100 r_g)

Therefore, $v_\phi = r\Omega_K, \quad \Omega_K = \sqrt{\frac{GM}{r^3}}$ is a good approximation

Accretion disk

Gas in the disk follows (almost) Keplerian orbit

Check :

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{\nabla P}{\rho} - \nabla \Phi_G \qquad \frac{\nabla P}{\rho} \sim \frac{(nk_B T)/r}{nm_p}, \quad \nabla \Phi_G \sim \frac{GM}{r^2}$$

gravitational force / pressure gradient $\sim \dfrac{GMm_p/r}{k_B T}$ *~ 500*
(1 M_sun BH accretor, T = 10keV, r ~ 100 r_g)

Therefore, $v_\phi = r\Omega_K, \quad \Omega_K = \sqrt{\dfrac{GM}{r^3}}$ is a good approximation

Conclusion : disk is differentially rotating with $\Omega \propto r^{-3/2}$

Angular momentum

For accertion, angular momentum of gas should be removed

Angular momentum

For accertion, angular momentum of gas should be removed

$$h = r^2\Omega_K = \sqrt{GMr} \propto r^{1/2}$$

Angular momentum

For accertion, angular momentum of gas should be removed

$$h = r^2\Omega_K = \sqrt{GMr} \propto r^{1/2}$$

Rayleigh stability criterion: $\frac{\partial h}{\partial r} > 0$

Angular momentum

For accertion, angular momentum of gas should be removed

$$h = r^2 \Omega_K = \sqrt{GMr} \propto r^{1/2}$$

Rayleigh stability criterion: $\frac{\partial h}{\partial r} > 0$ Keplerian disk satisfies this condition

Angular momentum

For accertion, angular momentum of gas should be removed

$$h = r^2\Omega_K = \sqrt{GMr} \propto r^{1/2}$$

Rayleigh stability criterion: $\dfrac{\partial h}{\partial r} > 0$ Keplerian disk satisfies this condition

Something should drive dissipative process that makes gas to fall inside

Angular momentum

For accertion, angular momentum of gas should be removed

$$h = r^2\Omega_K = \sqrt{GMr} \propto r^{1/2}$$

Rayleigh stability criterion: $\dfrac{\partial h}{\partial r} > 0$ Keplerian disk satisfies this condition

Something should drive dissipative process that makes gas to fall inside

Problem : molecular viscosity is too weak for that

Angular momentum

For accertion, angular momentum of gas should be removed

$$h = r^2\Omega_K = \sqrt{GMr} \propto r^{1/2}$$

Rayleigh stability criterion: $\dfrac{\partial h}{\partial r} > 0$ Keplerian disk satisfies this condition

Something should drive dissipative process that makes gas to fall inside

Problem : molecular viscosity is too weak for that

- **Shakura & Sunyaev (1973)** : turbulence as the source of increased viscosity

Shakura-Sunyaev disk model

Astron. & Astrophys. 24, 337 – 355 (1973)

Black Holes in Binary Systems. Observational Appearance

N. I. Shakura
Sternberg Astronomical Institute, Moscow, U.S.S.R.

R. A. Sunyaev
Institute of Applied Mathematics, Academy of Sciences, Moscow, U.S.S.R.

Received June 6, 1972

Shakura-Sunyaev disk model

Astron. & Astrophys. 24, 337 – 355 (1973)

Black Holes in Binary Systems. Observational Appearance

N. I. Shakura
Sternberg Astronomical Institute, Moscow, U.S.S.R.

R. A. Sunyaev
Institute of Applied Mathematics, Academy of Sciences, Moscow, U.S.S.R.

Received June 6, 1972

What is it? (what is the paper about?)

Shakura-Sunyaev disk model

Astron. & Astrophys. 24, 337–355 (1973)

Black Holes in Binary Systems. Observational Appearance

N. I. Shakura
Sternberg Astronomical Institute, Moscow, U.S.S.R.

R. A. Sunyaev
Institute of Applied Mathematics, Academy of Sciences, Moscow, U.S.S.R.

Received June 6, 1972

What is it? (what is the paper about?)

- analytic model of thin accretion disk

Shakura-Sunyaev disk model

Astron. & Astrophys. 24, 337—355 (1973)

Black Holes in Binary Systems. Observational Appearance

N. I. Shakura
Sternberg Astronomical Institute, Moscow, U.S.S.R.

R. A. Sunyaev
Institute of Applied Mathematics, Academy of Sciences, Moscow, U.S.S.R.

Received June 6, 1972

What is it? (what is the paper about?)

- analytic model of thin accretion disk

Assumptions & Properties

Shakura-Sunyaev disk model

Astron. & Astrophys. 24, 337 – 355 (1973)

Black Holes in Binary Systems. Observational Appearance

N. I. Shakura
Sternberg Astronomical Institute, Moscow, U.S.S.R.

R. A. Sunyaev
Institute of Applied Mathematics, Academy of Sciences, Moscow, U.S.S.R.

Received June 6, 1972

What is it? (what is the paper about?)

- analytic model of thin accretion disk

Assumptions & Properties

- stationary

Shakura-Sunyaev disk model

Astron. & Astrophys. 24, 337 – 355 (1973)

Black Holes in Binary Systems. Observational Appearance

N. I. Shakura
Sternberg Astronomical Institute, Moscow, U.S.S.R.

R. A. Sunyaev
Institute of Applied Mathematics, Academy of Sciences, Moscow, U.S.S.R.

Received June 6, 1972

What is it? (what is the paper about?)

- analytic model of thin accretion disk

Assumptions & Properties

- stationary
- axially symmetric

Shakura-Sunyaev disk model

Astron. & Astrophys. 24, 337 – 355 (1973)

Black Holes in Binary Systems. Observational Appearance

N. I. Shakura
Sternberg Astronomical Institute, Moscow, U.S.S.R.

R. A. Sunyaev
Institute of Applied Mathematics, Academy of Sciences, Moscow, U.S.S.R.

Received June 6, 1972

What is it? (what is the paper about?)

- analytic model of thin accretion disk

Assumptions & Properties

- stationary
- axially symmetric
- geometrically thin $(H \ll r)$

Shakura-Sunyaev disk model

Astron. & Astrophys. 24, 337—355 (1973)

Black Holes in Binary Systems. Observational Appearance

N. I. Shakura
Sternberg Astronomical Institute, Moscow, U.S.S.R.

R. A. Sunyaev
Institute of Applied Mathematics, Academy of Sciences, Moscow, U.S.S.R.

Received June 6, 1972

What is it? (what is the paper about?)

- analytic model of thin accretion disk

Assumptions & Properties

- stationary
- axially symmetric
- geometrically thin $(H \ll r)$
- optically thick

Shakura-Sunyaev disk model

Astron. & Astrophys. 24, 337 – 355 (1973)

Black Holes in Binary Systems. Observational Appearance

N. I. Shakura
Sternberg Astronomical Institute, Moscow, U.S.S.R.

R. A. Sunyaev
Institute of Applied Mathematics, Academy of Sciences, Moscow, U.S.S.R.

Received June 6, 1972

What is it? (what is the paper about?)

- analytic model of thin accretion disk

Assumptions & Properties

- stationary
- axially symmetric
- geometrically thin $(H \ll r)$
- optically thick
- radiation-efficient (all heats are radiated away)

Shakura-Sunyaev disk model

Astron. & Astrophys. 24, 337 — 355 (1973)

Black Holes in Binary Systems. Observational Appearance

N. I. Shakura
Sternberg Astronomical Institute, Moscow, U.S.S.R.

R. A. Sunyaev
Institute of Applied Mathematics, Academy of Sciences, Moscow, U.S.S.R.

Received June 6, 1972

What is it? (what is the paper about?)

- analytic model of thin accretion disk

Assumptions & Properties

- stationary
- axially symmetric
- geometrically thin $(H \ll r)$
- optically thick
- radiation-efficient (all heats are radiated away)
- <u>parametrization of viscous effect with a parameter</u> α

Derivation (high level overview)

Derivation (high level overview)

$$\mathbf{v} = (v_r, v_\phi, v_z) \qquad v_\phi = r\Omega_K = \sqrt{GM/r}, \quad v_r \ll v_\phi, \quad v_z = 0$$

Derivation (high level overview)

$$\mathbf{v} = (v_r, v_\phi, v_z) \qquad v_\phi = r\Omega_K = \sqrt{GM/r}, \quad v_r \ll v_\phi, \quad v_z = 0$$

For a thin disk, it is useful to define $\Sigma(r) = \int \rho dz$

Derivation (high level overview)

$$\mathbf{v} = (v_r, v_\phi, v_z) \qquad v_\phi = r\Omega_K = \sqrt{GM/r}, \quad v_r \ll v_\phi, \quad v_z = 0$$

For a thin disk, it is useful to define $\Sigma(r) = \int \rho dz$

- Mass conservation: $\dot{M} \equiv -2\pi r \Sigma v_r = \text{const}$

Derivation (high level overview)

$$\mathbf{v} = (v_r, v_\phi, v_z) \qquad v_\phi = r\Omega_K = \sqrt{GM/r}, \quad v_r \ll v_\phi, \quad v_z = 0$$

For a thin disk, it is useful to define $\Sigma(r) = \int \rho dz$

- Mass conservation: $\dot{M} \equiv -2\pi r \Sigma v_r = \text{const}$
- Momentum equation with viscosity : $\nu\Sigma = \dfrac{\dot{M}}{3\pi}\left(1 - \sqrt{\dfrac{r_0}{r}}\right)$

Derivation (high level overview)

$$\mathbf{v} = (v_r, v_\phi, v_z) \qquad v_\phi = r\Omega_K = \sqrt{GM/r}, \quad v_r \ll v_\phi, \quad v_z = 0$$

For a thin disk, it is useful to define $\Sigma(r) = \int \rho dz$

- Mass conservation: $\dot{M} \equiv -2\pi r \Sigma v_r = \text{const}$
- Momentum equation with viscosity : $\nu\Sigma = \frac{\dot{M}}{3\pi}\left(1 - \sqrt{\frac{r_0}{r}}\right)$

r_0 : inner disk boundary
where torque vanishes

Derivation (high level overview)

$$\mathbf{v} = (v_r, v_\phi, v_z) \qquad v_\phi = r\Omega_K = \sqrt{GM/r}, \quad v_r \ll v_\phi, \quad v_z = 0$$

For a thin disk, it is useful to define $\Sigma(r) = \int \rho dz$

- Mass conservation: $\dot{M} \equiv -2\pi r \Sigma v_r = \text{const}$
- Momentum equation with viscosity : $\nu\Sigma = \frac{\dot{M}}{3\pi}\left(1 - \sqrt{\frac{r_0}{r}}\right)$

r_0 : inner disk boundary where torque vanishes

alpha-viscosity prescription : $\nu = \alpha c_s H = \alpha c_s^2 / \Omega_K$

Derivation (high level overview)

$$\mathbf{v} = (v_r, v_\phi, v_z) \qquad v_\phi = r\Omega_K = \sqrt{GM/r}, \quad v_r \ll v_\phi, \quad v_z = 0$$

For a thin disk, it is useful to define $\Sigma(r) = \int \rho dz$

- Mass conservation: $\dot{M} \equiv -2\pi r \Sigma v_r = \text{const}$
- Momentum equation with viscosity : $\nu\Sigma = \dfrac{\dot{M}}{3\pi}\left(1 - \sqrt{\dfrac{r_0}{r}}\right)$

r_0 : inner disk boundary where torque vanishes

alpha-viscosity prescription : $\nu = \alpha c_s H = \alpha c_s^2/\Omega_K$

recall) $\nu \sim \langle v \rangle \lambda_{\rm mfp}$ $\qquad \alpha = \dfrac{\nu}{c_s H} = \dfrac{\langle v \rangle}{c_s}\dfrac{\lambda_{\rm mfp}}{H} \leq 1$

Derivation (high level overview)

$$\mathbf{v} = (v_r, v_\phi, v_z) \qquad v_\phi = r\Omega_K = \sqrt{GM/r}, \quad v_r \ll v_\phi, \quad v_z = 0$$

For a thin disk, it is useful to define $\Sigma(r) = \int \rho dz$

- Mass conservation: $\dot{M} \equiv -2\pi r \Sigma v_r = \text{const}$
- Momentum equation with viscosity : $\nu\Sigma = \dfrac{\dot{M}}{3\pi}\left(1 - \sqrt{\dfrac{r_0}{r}}\right)$

r_0 : inner disk boundary where torque vanishes

alpha-viscosity prescription : $\nu = \alpha c_s H = \alpha c_s^2 / \Omega_K$

recall) $\nu \sim \langle v \rangle \lambda_{\rm mfp}$ $\qquad \alpha = \dfrac{\nu}{c_s H} = \dfrac{\langle v \rangle}{c_s}\dfrac{\lambda_{\rm mfp}}{H} \leq 1$

- Energy equation : $Q = \dfrac{3}{8\pi}\dot{M}\dfrac{GM}{R^3}\left\{1 - \left(\dfrac{R_0}{R}\right)^{1/2}\right\}.$

Equating heat production with radiation would determine thermodynamic properties

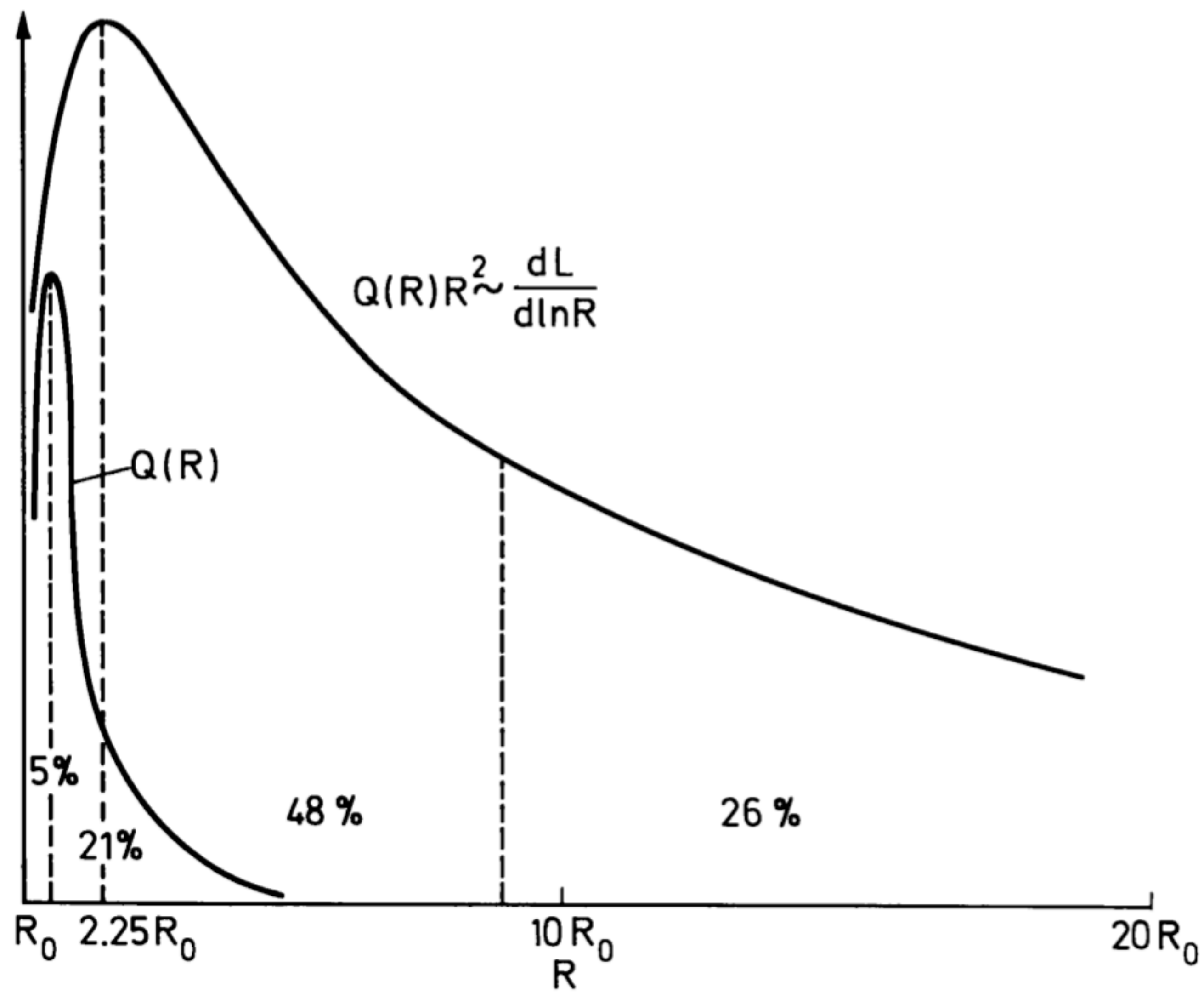


Fig. 7. The luminosity of the surface unit of the disk as a function of the radius. The function $Q(R)\,R^2$ is proportional to the luminosity of the ring with radius R and $\Delta R \sim R$. The numbers illustrate the contribution of the corresponding regions to the integral luminosity of the disk

Solutions

Piecewise solutions over three regimes:

Solutions

Piecewise solutions over three regimes:

a) radiation pressure dominant, electron scattering opacity

$$z_0[\mathrm{cm}] = \frac{3}{8\pi}\frac{\sigma_T}{c}\dot{M}(1-r^{-1/2}) = 3.2\cdot 10^6 \dot{m}\mathrm{m}\,(1-z^{-1/2}) \tag{2.8}$$

$$u_0\left[\frac{\mathrm{g}}{\mathrm{cm}^2}\right] = \frac{64\pi}{9\alpha}\frac{c^2}{\sigma^2}\frac{1}{\omega\dot{M}(1-r^{-1/2})} = 4.6\alpha^{-1}\dot{m}^{-1}r^{3/2}(1-r^{-1/2})^{-1}\,, \tag{2.9}$$

$$\varepsilon\left[\frac{\mathrm{erg}}{\mathrm{cm}^3}\right] = 2\frac{c}{\sigma}\omega = 2.1\cdot 10^{15}\alpha^{-1}m^{-1}r^{-3/2}\,, \tag{2.10}$$

$$\left.\begin{aligned} n[\mathrm{cm}^{-3}] &= \frac{u_0}{2m_p z_0} \\ &= 4.3\cdot 10^{17}\alpha^{-1}\dot{m}^{-2}m^{-1}r^{3/2}(1-r^{-1/2})^{-2} \\ v_r\left[\frac{\mathrm{cm}}{\mathrm{s}}\right] &= \frac{\dot{M}}{2\pi u_0 R} \\ &= 7.7\cdot 10^{10}\alpha\dot{m}^2 r^{-5/2}(1-r^{-1/2}) \\ H[\mathrm{Gauss}] &\leqq \sqrt{\frac{4\pi}{3}\alpha\varepsilon} = 10^8 m^{-1/2}r^{-3/4}\,. \end{aligned}\right\} \tag{2.11}$$

$$T = 2.3\cdot 10^7(\alpha m)^{-1/4}r^{-3/4}\,{}^\circ\mathrm{K}\,. \tag{2.12}$$

$$\tau^* = 8.4\cdot 10^{-5}\alpha^{-17/16}m^{-1/16}\dot{m}^{-2}\cdot r^{-93/32}(1-r^{-1/2})^{-2}\,.$$

b) matter pressure dominant, electron scattering opacity

b) $P_g \gg P_r, \quad \sigma_T \gg \sigma_{ff}$

$$u_0 = 1.7 \cdot 10^5 \alpha^{-4/5} \dot{m}^{3/5} m^{1/5} r^{-3/5} (1 - r^{-1/2})^{3/5}$$
$$T = 3.1 \cdot 10^8 \alpha^{-1/5} \dot{m}^{2/5} m^{-1/5} r^{-9/10} (1 - r^{-1/2})^{2/5}$$
$$z_0 = 1.2 \cdot 10^4 \alpha^{-1/10} \dot{m}^{1/5} m^{9/10} r^{21/20} (1 - r^{-1/2})^{1/5} \quad (2.16)$$
$$n = 4.2 \cdot 10^{24} \alpha^{-7/10} \dot{m}^{2/5} m^{-7/10} r^{-33/20} (1 - r^{-1/2})^{2/5}$$
$$\tau^* = \sqrt{\sigma_{ff} \sigma_T} u_0 = 10^2 \alpha^{-4/5} \dot{m}^{9/10} m^{1/5} r^{3/20} (1 - r^{-1/2})^{9/10}$$
$$v_r = 2 \cdot 10^6 \alpha^{4/5} \dot{m}^{2/5} m^{-1/5} r^{-2/5} (1 - r^{-1/2})^{-3/5}$$
$$H \leqq 1.5 \cdot 10^9 \alpha^{1/20} \dot{m}^{2/5} m^{-9/20} r^{-51/40} (1 - r^{-1/2})^{2/5} .$$

$$\frac{r_{ab}}{(1 - r_{ab}^{-1/2})^{16/21}} = 150 (\alpha m)^{2/21} \dot{m}^{16/21}$$

b) matter pressure dominant, electron scattering opacity

b) $P_g \gg P_r, \quad \sigma_T \gg \sigma_{ff}$

$$
\begin{aligned}
u_0 &= 1.7 \cdot 10^5 \alpha^{-4/5} \dot{m}^{3/5} m^{1/5} r^{-3/5} (1 - r^{-1/2})^{3/5} \\
T &= 3.1 \cdot 10^8 \alpha^{-1/5} \dot{m}^{2/5} m^{-1/5} r^{-9/10} (1 - r^{-1/2})^{2/5} \\
z_0 &= 1.2 \cdot 10^4 \alpha^{-1/10} \dot{m}^{1/5} m^{9/10} r^{21/20} (1 - r^{-1/2})^{1/5} \qquad (2.16) \\
n &= 4.2 \cdot 10^{24} \alpha^{-7/10} \dot{m}^{2/5} m^{-7/10} r^{-33/20} (1 - r^{-1/2})^{2/5} \\
\tau^* &= \sqrt{\sigma_{ff}\sigma_T}\, u_0 = 10^2 \alpha^{-4/5} \dot{m}^{9/10} m^{1/5} r^{3/20} (1 - r^{-1/2})^{9/10} \\
v_r &= 2 \cdot 10^6 \alpha^{4/5} \dot{m}^{2/5} m^{-1/5} r^{-2/5} (1 - r^{-1/2})^{-3/5} \\
H &\lesseqqgtr 1.5 \cdot 10^9 \alpha^{1/20} \dot{m}^{2/5} m^{-9/20} r^{-51/40} (1 - r^{-1/2})^{2/5} .
\end{aligned}
$$

$$\frac{r_{ab}}{(1 - r_{ab}^{-1/2})^{16/21}} = 150 (\alpha m)^{2/21} \dot{m}^{16/21}$$

c) matter pressure dominant, free-free opacity

c) $P_r \ll P_g, \quad \sigma_{ff} \gg \sigma_T$

$$
\begin{aligned}
u_0 &= 6.1 \cdot 10^5 \alpha^{-4/5} \dot{m}^{7/10} m^{1/5} r^{-3/4} (1 - r^{-1/2})^{7/10} \\
T &= 8.6 \cdot 10^7 \alpha^{-1/5} \dot{m}^{3/10} m^{-1/5} r^{-3/4} (1 - z^{-1/2})^{3/10} \\
z_0 &= 6.1 \cdot 10^3 \alpha^{-1/10} \dot{m}^{3/20} m^{9/10} r^{9/8} (1 - r^{-1/2})^{3/20} \qquad (2.19) \\
n &= 3 \cdot 10^{25} \alpha^{-7/10} \dot{m}^{11/20} m^{-7/10} r^{-15/8} (1 - r^{-1/2})^{11/20} \\
\tau &= \sigma_{ff} u_0 = 3.4 \cdot 10^2 \alpha^{-4/5} \dot{m}^{1/5} m^{1/5} (1 - r^{-1/2})^{1/5} \\
v_r &= 5.8 \cdot 10^5 \alpha^{4/5} \dot{m}^{3/10} m^{-1/5} r^{-1/4} (1 - r^{-1/2})^{-7/10} \\
H &\lesssim 2.1 \cdot 10^9 \alpha^{1/20} \dot{m}^{17/40} m^{-9/20} r^{-21/16} (1 - r^{-1/2})^{17/40}
\end{aligned}
$$

$$r_{bc} = 6.3 \cdot 10^3 \dot{m}^{2/3} (1 - r_{bc}^{-1/2})^{2/3}$$

b) matter pressure dominant, electron scattering opacity

b) $P_g \gg P_r, \quad \sigma_T \gg \sigma_{ff}$

$$u_0 = 1.7 \cdot 10^5 \alpha^{-4/5} \dot{m}^{3/5} m^{1/5} r^{-3/5} (1 - r^{-1/2})^{3/5}$$
$$T = 3.1 \cdot 10^8 \alpha^{-1/5} \dot{m}^{2/5} m^{-1/5} r^{-9/10} (1 - r^{-1/2})^{2/5}$$
$$z_0 = 1.2 \cdot 10^4 \alpha^{-1/10} \dot{m}^{1/5} m^{9/10} r^{21/20} (1 - r^{-1/2})^{1/5} \quad (2.16)$$
$$n = 4.2 \cdot 10^{24} \alpha^{-7/10} \dot{m}^{2/5} m^{-7/10} r^{-33/20} (1 - r^{-1/2})^{2/5}$$
$$\tau^* = \sqrt{\sigma_{ff} \sigma_T}\, u_0 = 10^2 \alpha^{-4/5} \dot{m}^{9/10} m^{1/5} r^{3/20} (1 - r^{-1/2})^{9/10}$$
$$v_r = 2 \cdot 10^6 \alpha^{4/5} \dot{m}^{2/5} m^{-1/5} r^{-2/5} (1 - r^{-1/2})^{-3/5}$$
$$H \leqq 1.5 \cdot 10^9 \alpha^{1/20} \dot{m}^{2/5} m^{-9/20} r^{-51/40} (1 - r^{-1/2})^{2/5} .$$

$$\frac{r_{ab}}{(1 - r_{ab}^{-1/2})^{16/21}} = 150 (\alpha m)^{2/21} \dot{m}^{16/21}$$

↑

transition radii mostly dependent on the accretion rate

↓

$$r_{bc} = 6.3 \cdot 10^3 \dot{m}^{2/3} (1 - r_{bc}^{-1/2})^{2/3}$$

c) matter pressure dominant, free-free opacity

c) $P_r \ll P_g, \quad \sigma_{ff} \gg \sigma_T$

$$u_0 = 6.1 \cdot 10^5 \alpha^{-4/5} \dot{m}^{7/10} m^{1/5} r^{-3/4} (1 - r^{-1/2})^{7/10}$$
$$T = 8.6 \cdot 10^7 \alpha^{-1/5} \dot{m}^{3/10} m^{-1/5} r^{-3/4} (1 - z^{-1/2})^{3/10}$$
$$z_0 = 6.1 \cdot 10^3 \alpha^{-1/10} \dot{m}^{3/20} m^{9/10} r^{9/8} (1 - r^{-1/2})^{3/20} \quad (2.19)$$
$$n = 3 \cdot 10^{25} \alpha^{-7/10} \dot{m}^{11/20} m^{-7/10} r^{-15/8} (1 - r^{-1/2})^{11/20}$$
$$\tau = \sigma_{ff} u_0 = 3.4 \cdot 10^2 \alpha^{-4/5} \dot{m}^{1/5} m^{1/5} (1 - r^{-1/2})^{1/5}$$
$$v_r = 5.8 \cdot 10^5 \alpha^{4/5} \dot{m}^{3/10} m^{-1/5} r^{-1/4} (1 - r^{-1/2})^{-7/10}$$
$$H \lesssim 2.1 \cdot 10^9 \alpha^{1/20} \dot{m}^{17/40} m^{-9/20} r^{-21/16} (1 - r^{-1/2})^{17/40}$$

Note) solution depends on 3 parameters : $M, \dot{M}, \alpha$

For a fixed accertor, big picture of dynamics is determined by two paramters

$\dot{M}$ mass inflow from companion

α strength of viscosity

Note) solution depends on 3 parameters : $M, \dot{M}, \alpha$

For a fixed accertor, big picture of dynamics is determined by two paramters

$\dot{M}$ mass inflow from companion

α strength of viscosity

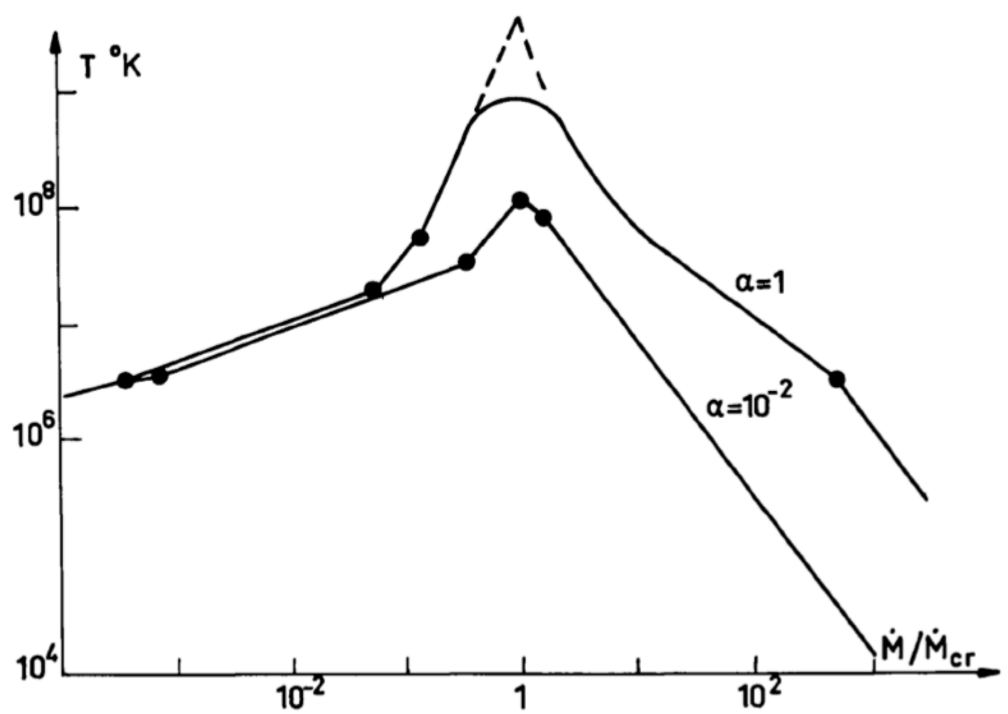


Fig. 6. Maximum effective temperature of radiation of the disk as a function of the flux of matter inflowing into it for different values of the efficiency α of the mechanisms of angular momentum transfer

Note) solution depends on 3 parameters : $M, \dot{M}, \alpha$

For a fixed accertor, big picture of dynamics is determined by two paramters

$\dot{M}$ mass inflow from companion

α strength of viscosity

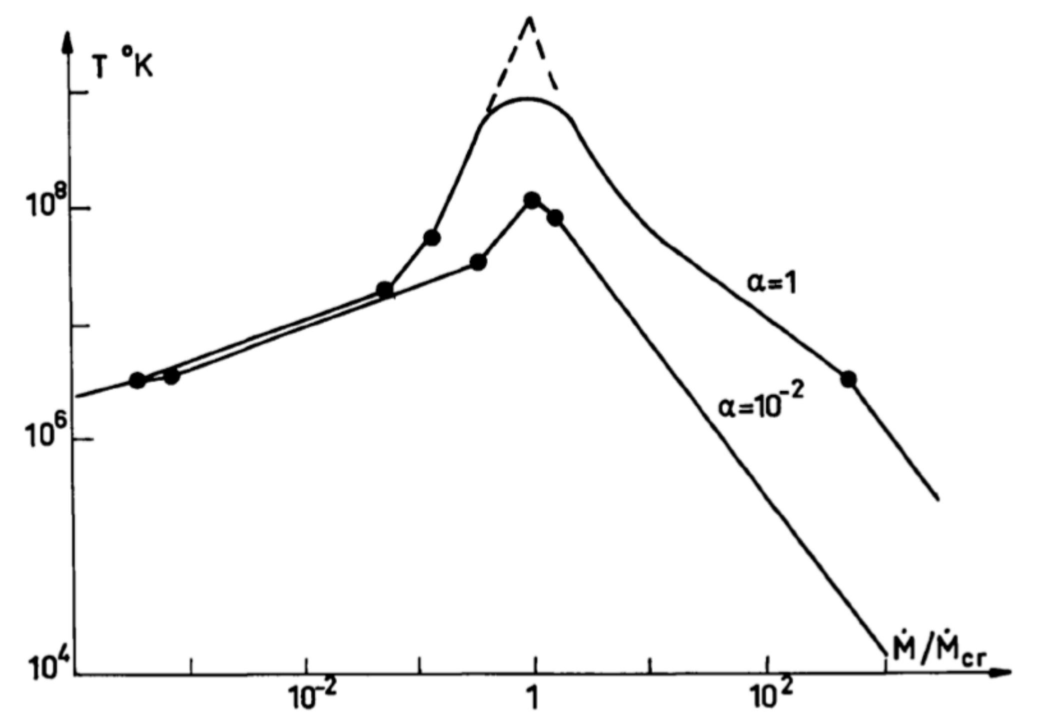
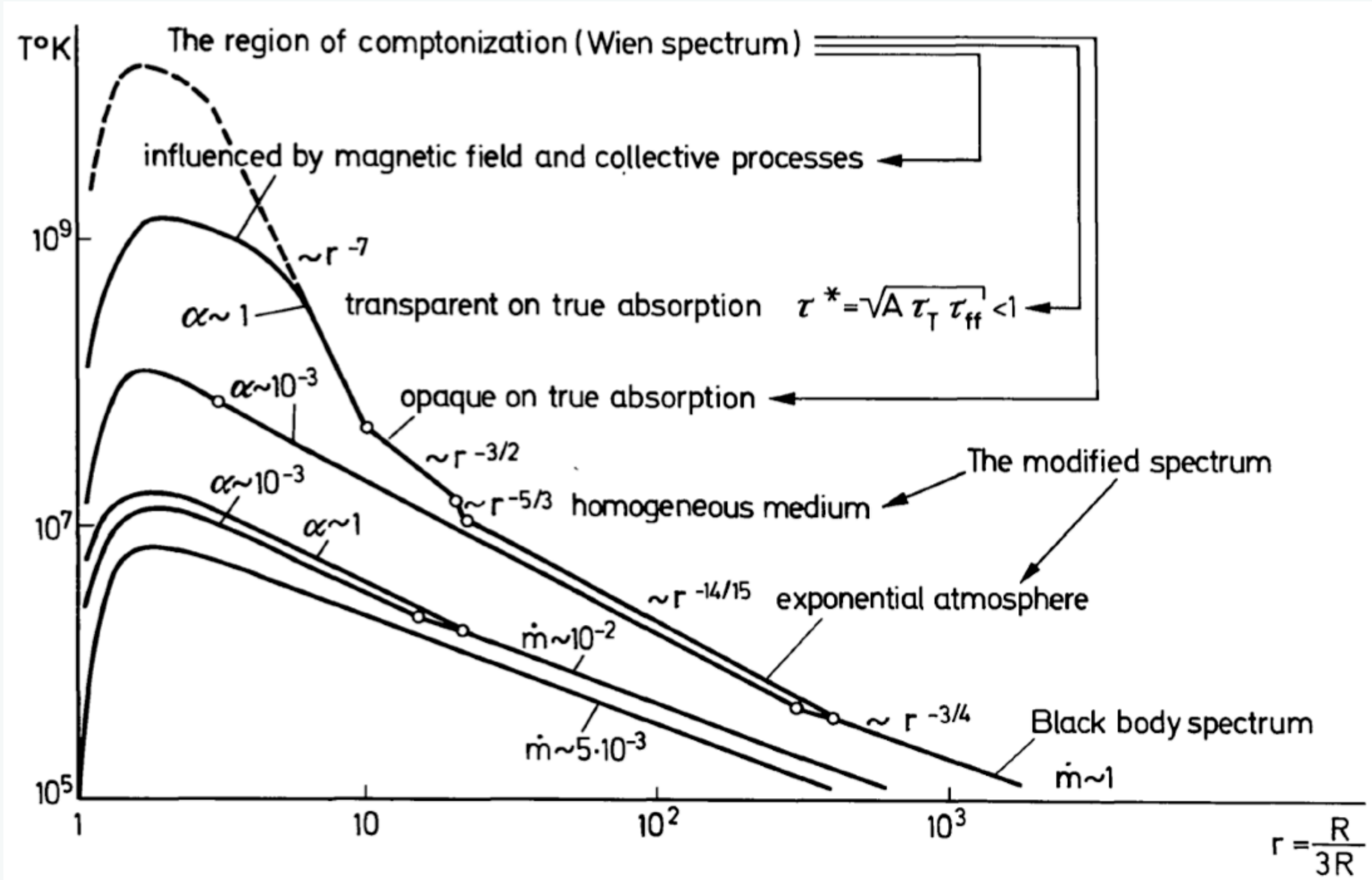


Fig. 6. Maximum effective temperature of radiation of the disk as a function of the flux of matter inflowing into it for different values of the efficiency α of the mechanisms of angular momentum transfer



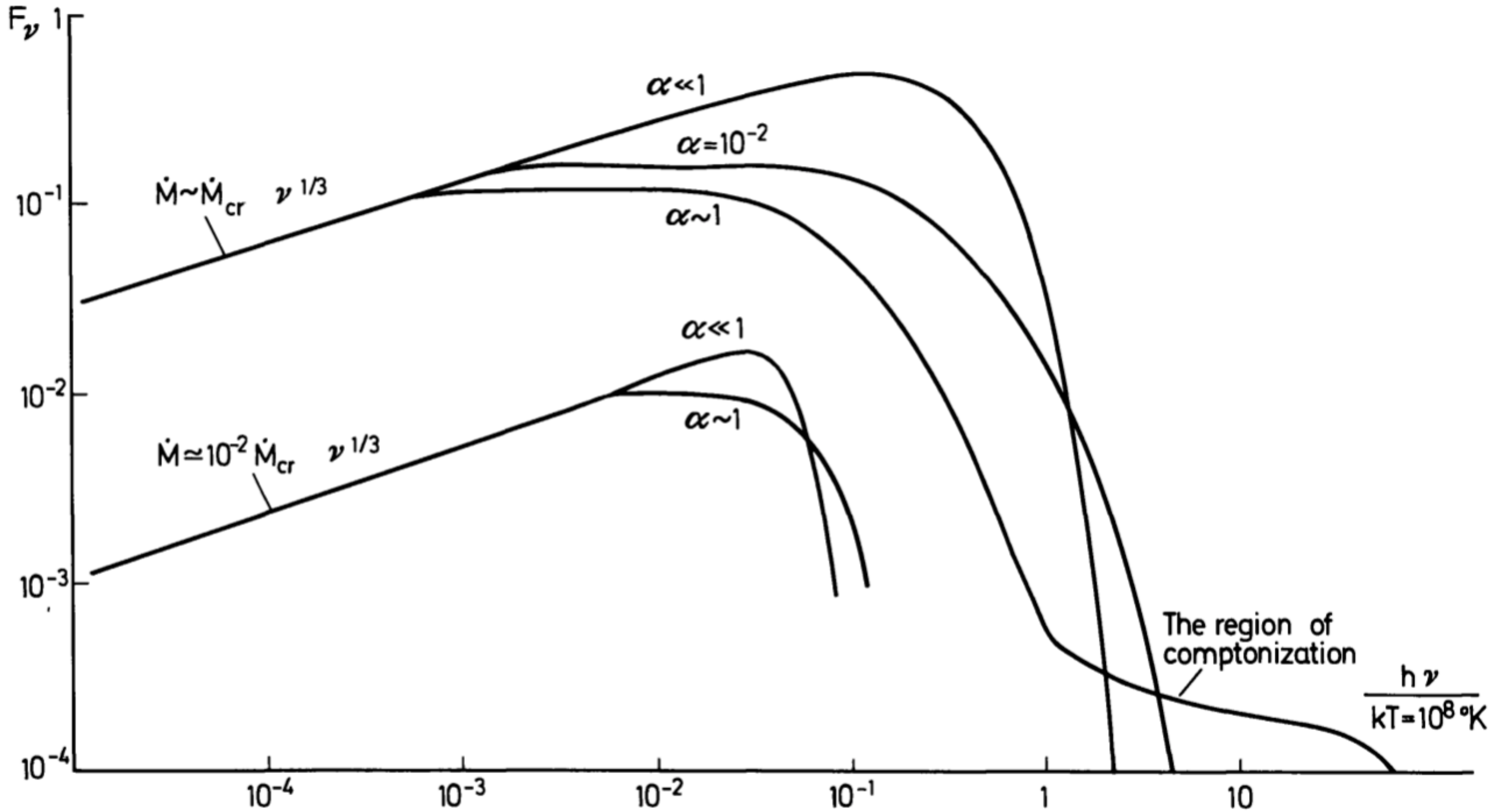
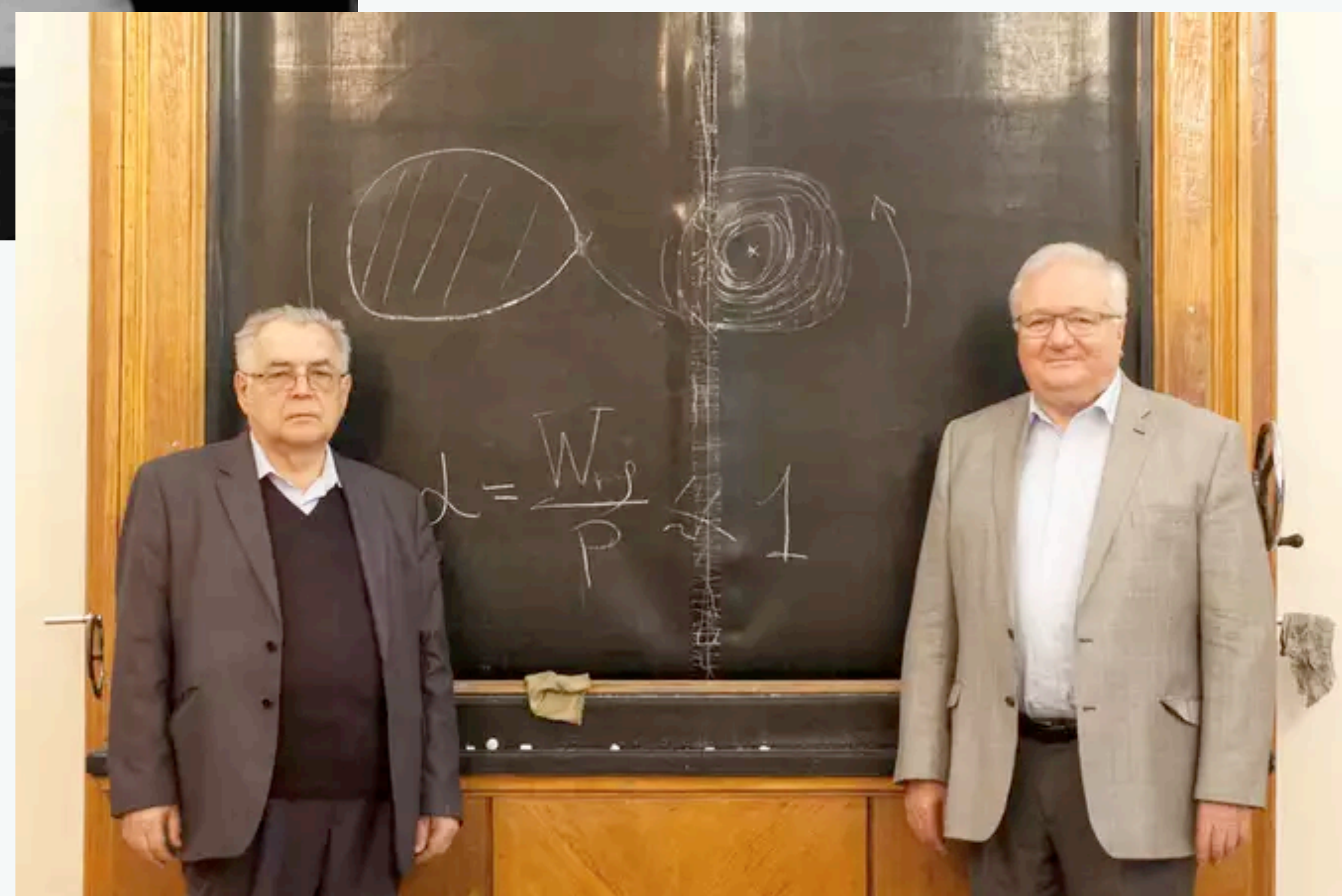


Fig. 3. The integral radiation spectrum of the disk, computed for different $\dot{M}$ and α

https://www.mpa-garching.mpg.de/454527/news20170619



https://www.mpa-garching.mpg.de/454527/news20170619

So, *what* drives turbulence in accretion disks?

So, *what* drives turbulence in accretion disks?

Lifelong advice from my undergrad advisor :

So, *what* drives turbulence in accretion disks?

Lifelong advice from my undergrad advisor :

In astronomy, if you see something strange,
it's mostly related to magnetic field…

Magneto-rotational Instability (MRI)

Balbus & Hawley (1991)

(firstly noticed by Velikhov in 1959)

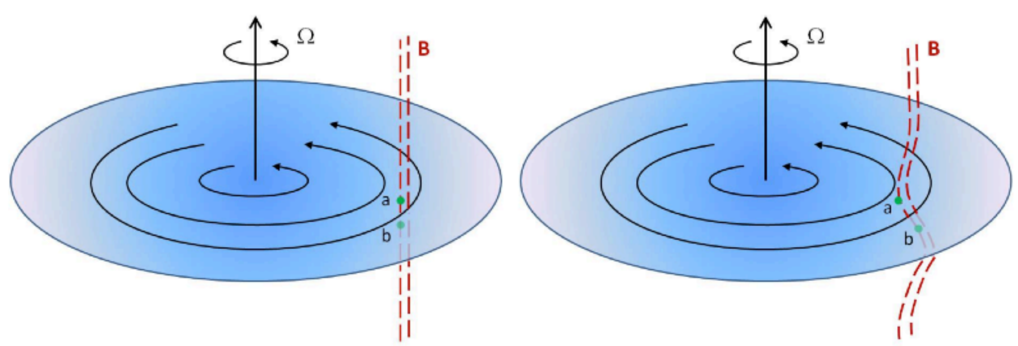


Figure 2: Magnetic field line deformation in the MRI instability. For a weak field, the braking of fluid element *a* pushed inward and outward acceleration of fluid element *b* will transfer angular momentum from *a* to *b*. The process continues by further stretching the distance of the two thereby causing an instability.

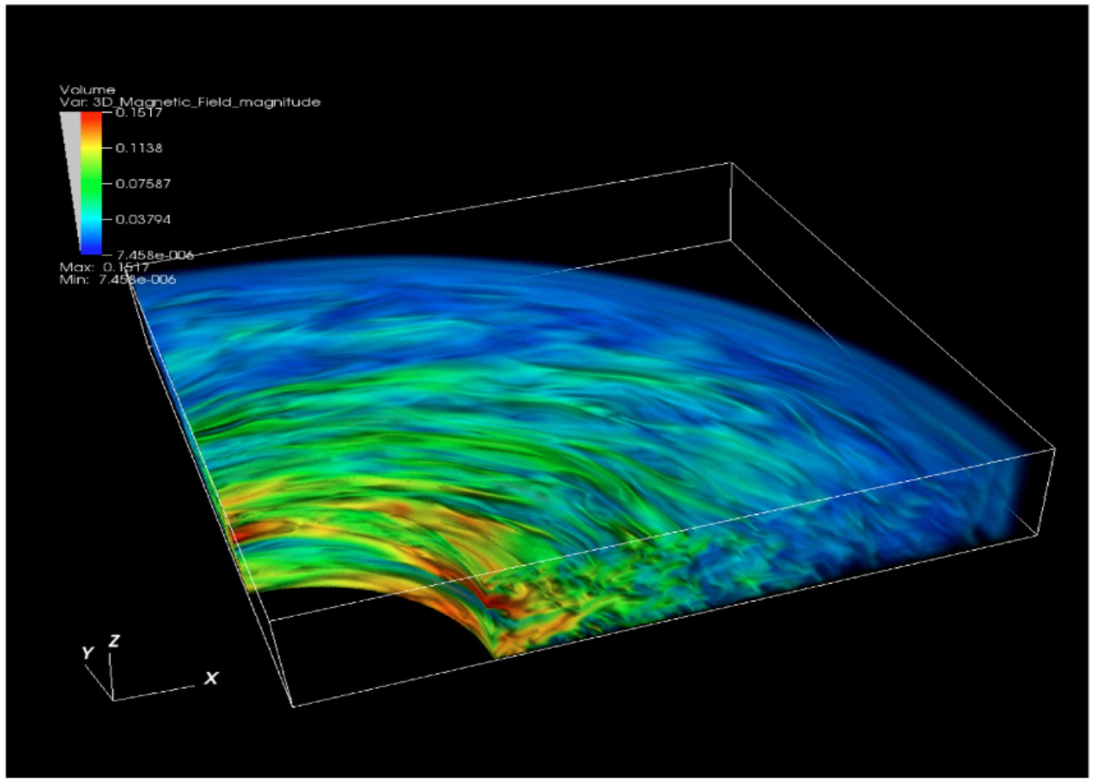


Figure 5: Disk section from a 3D numerical simulation of the MRI instability.

Magneto-rotational Instability (MRI)

Balbus & Hawley (1991)

(firstly noticed by Velikhov in 1959)

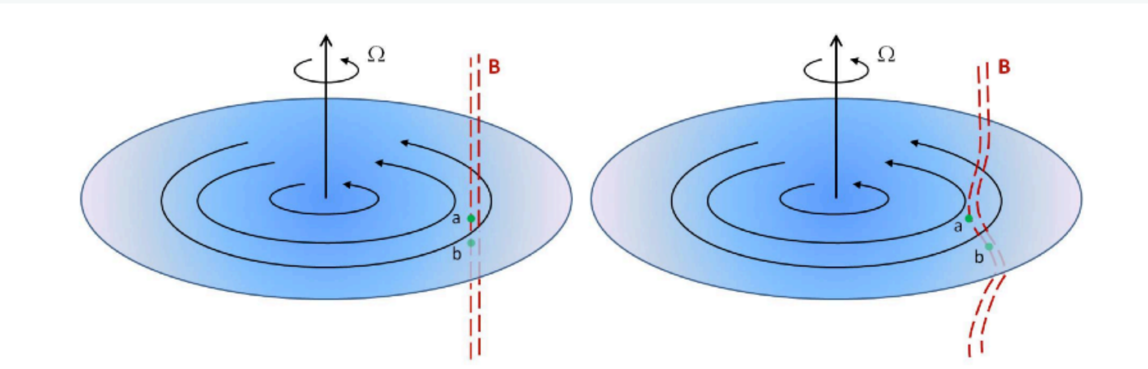


Figure 2: Magnetic field line deformation in the MRI instability. For a weak field, the braking of fluid element a pushed inward and outward acceleration of fluid element b will transfer angular momentum from a to b. The process continues by further stretching the distance of the two thereby causing an instability.

- Triggered when $\dfrac{d\Omega}{dr} < 0$

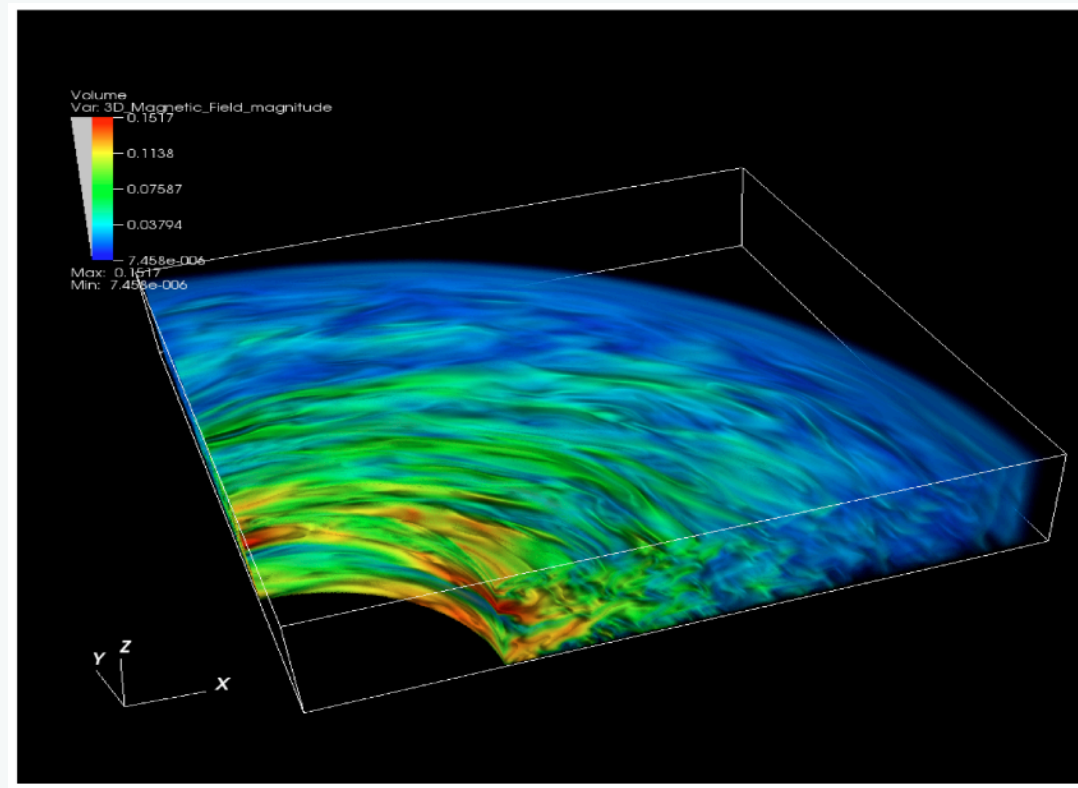


Figure 5: Disk section from a 3D numerical simulation of the MRI instability.

Magneto-rotational Instability (MRI)

Balbus & Hawley (1991)

(firstly noticed by Velikhov in 1959)

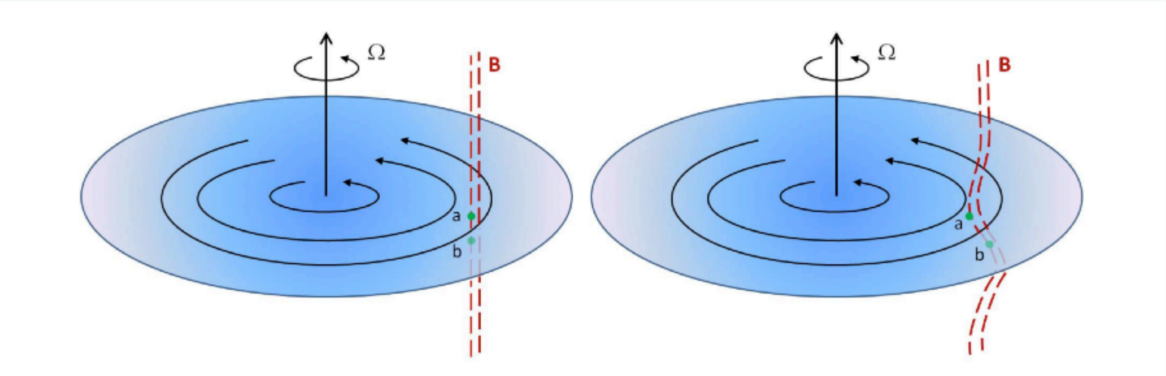


Figure 2: Magnetic field line deformation in the MRI instability. For a weak field, the braking of fluid element *a* pushed inward and outward acceleration of fluid element *b* will transfer angular momentum from *a* to *b*. The process continues by further stretching the distance of the two thereby causing an instability.

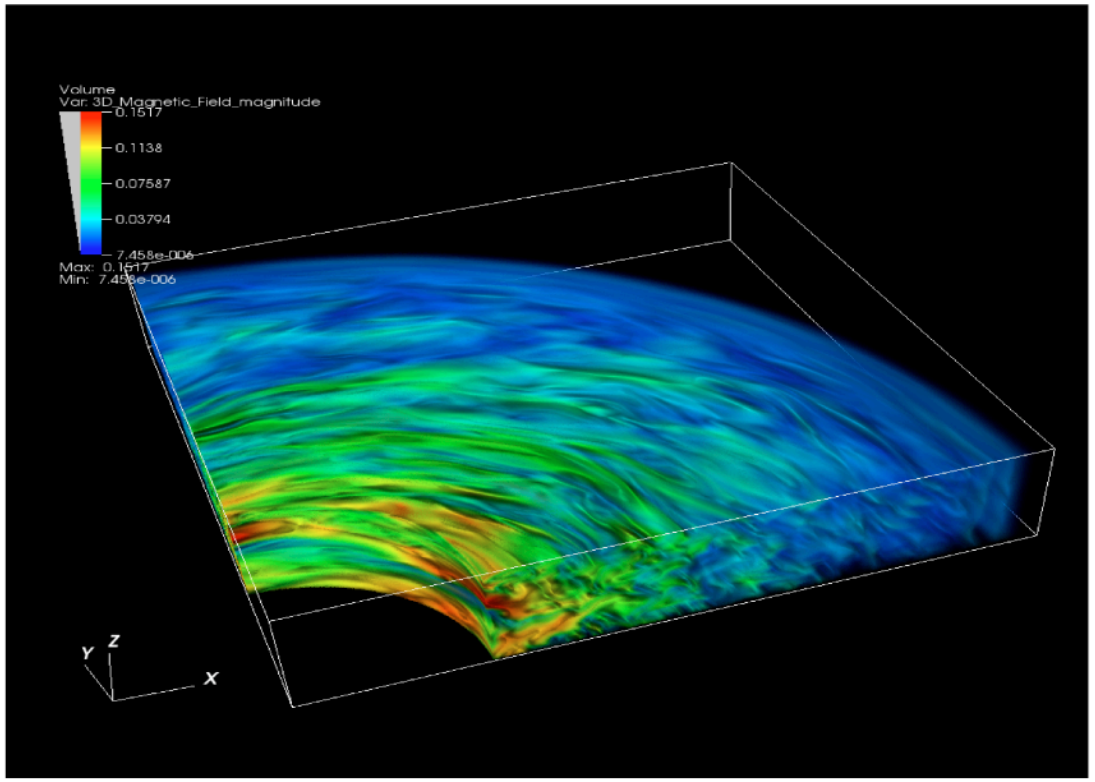


Figure 5: Disk section from a 3D numerical simulation of the MRI instability.

- Triggered when $\dfrac{d\Omega}{dr} < 0$
- sensitive to weak magnetic field

Magneto-rotational Instability (MRI)

Balbus & Hawley (1991)

(firstly noticed by Velikhov in 1959)

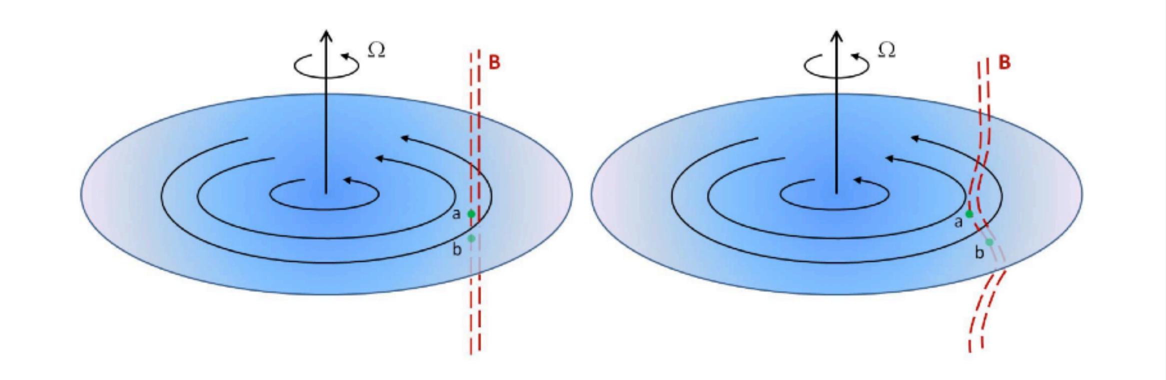


Figure 2: Magnetic field line deformation in the MRI instability. For a weak field, the braking of fluid element a pushed inward and outward acceleration of fluid element b will transfer angular momentum from a to b. The process continues by further stretching the distance of the two thereby causing an instability.

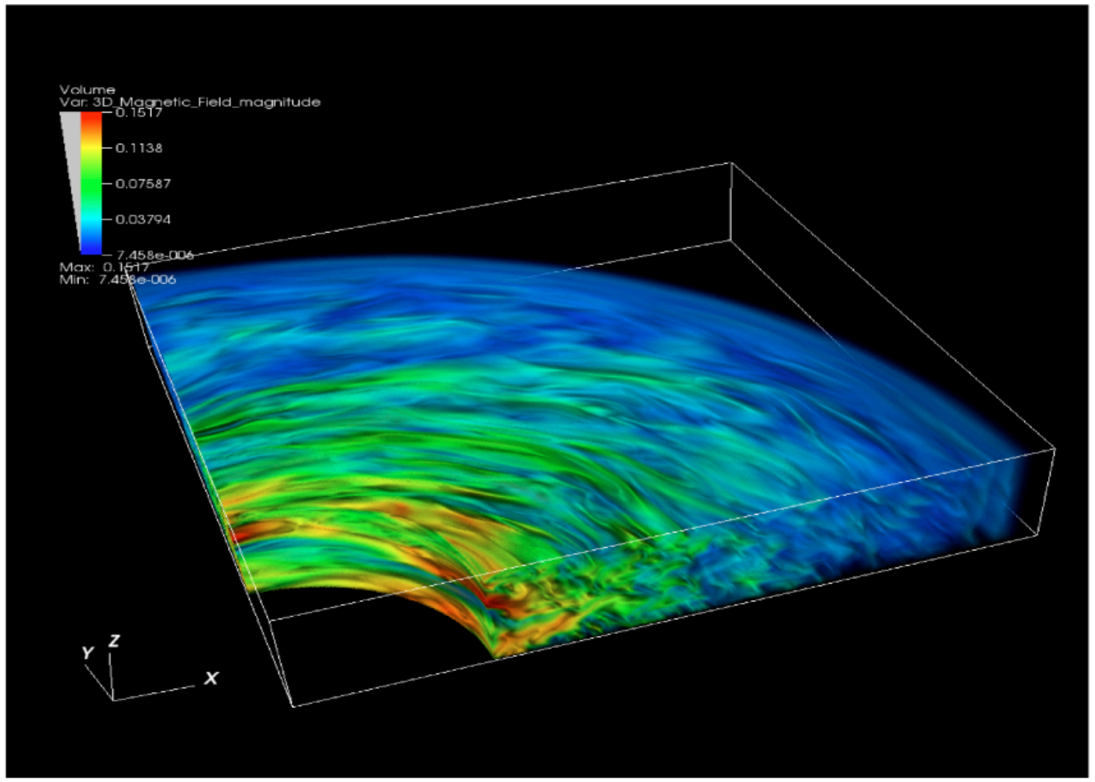
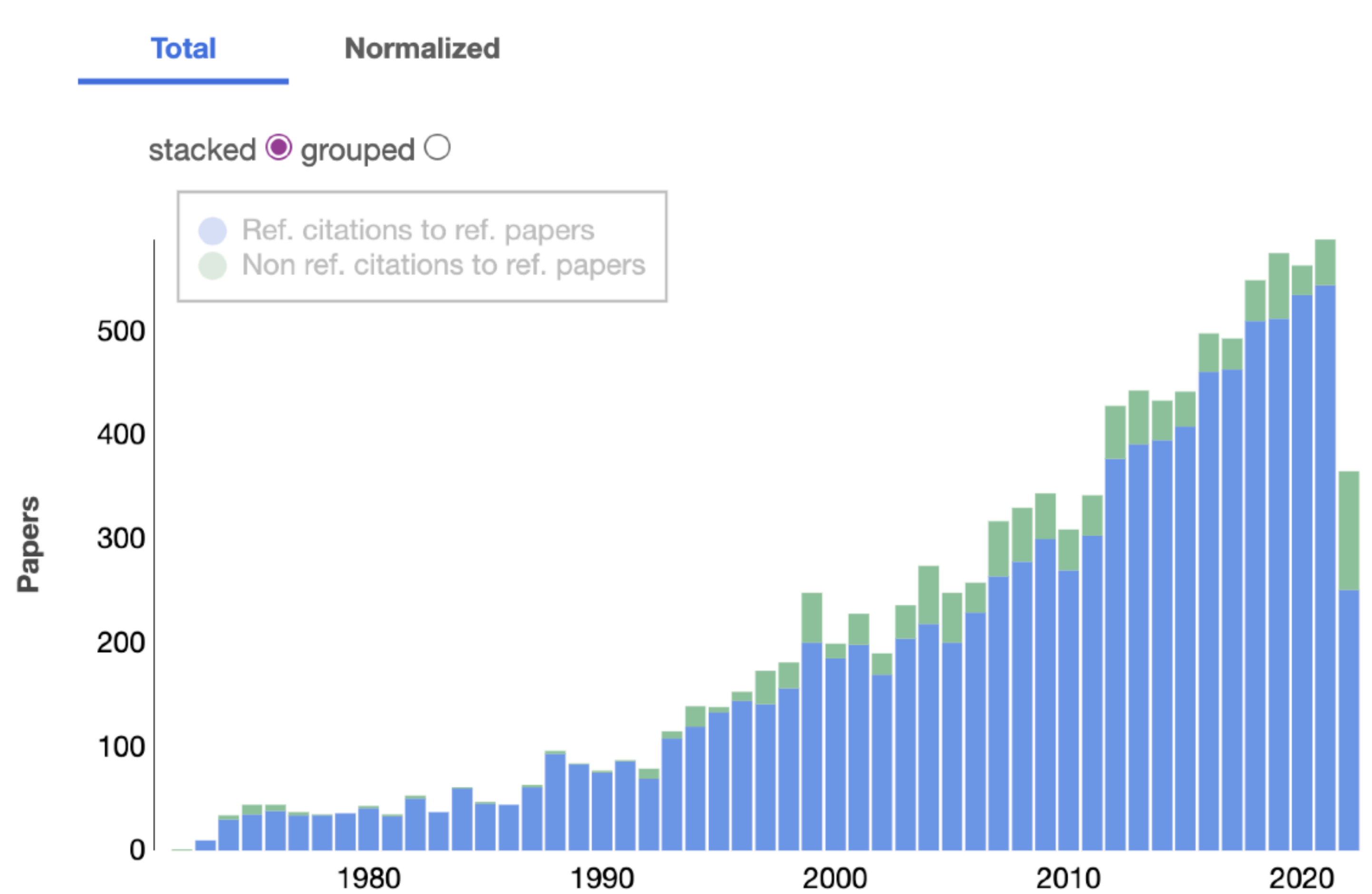


Figure 5: Disk section from a 3D numerical simulation of the MRI instability.

- Triggered when $\dfrac{d\Omega}{dr} < 0$
- sensitive to weak magnetic field
- important (but not the only) physical mechanism driving turbulence on astrophysical disks

- The alpha-disk model still finds its application today

Total citations	10846
Normalized citations	5423
Refereed citations	9660
Normalized refereed citations	4830



References

References

- Shakura, N. I. and Sunyaev, R. A. (1973)
  Black holes in binary systems. Observational appearance, Astronomy and Astrophysics, vol. 24, p. 337
- Balbus, Steven A.; Hawley, John F. (1991)
  A powerful local shear instability in weakly magnetized disks. I – Linear analysis. II – Nonlinear evolution, Astrophysical Journal, vol. 376, p. 214