Mass Transfer in Binaries

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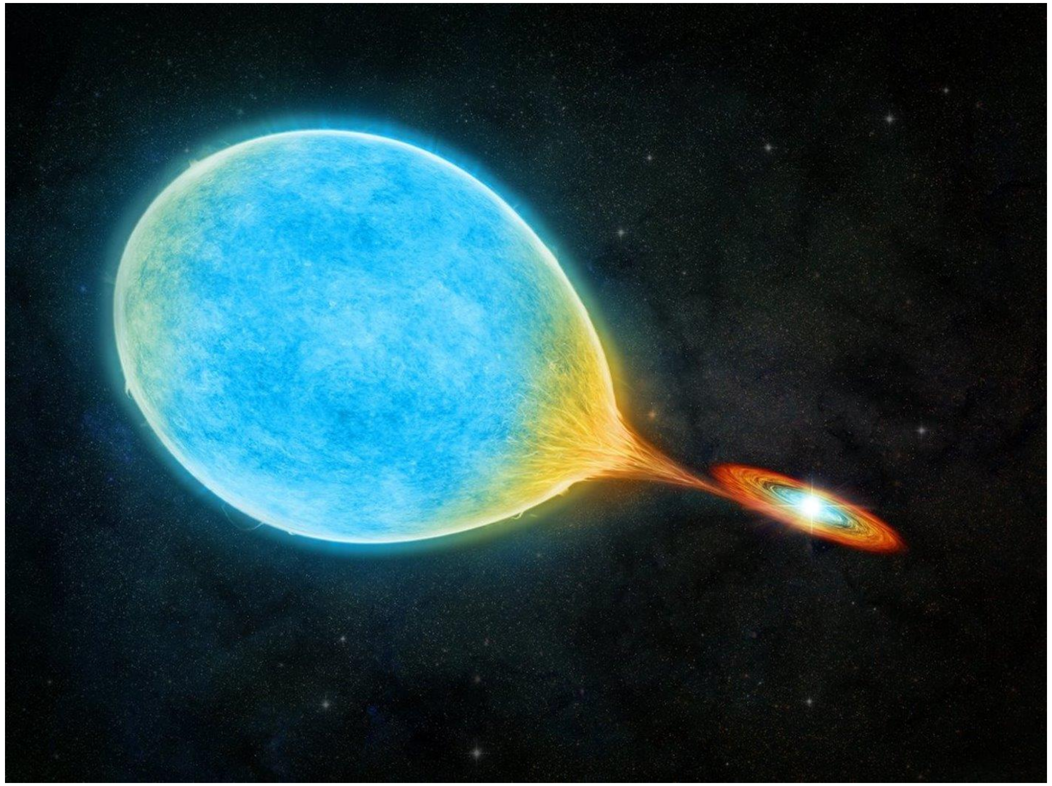
Background

- ≈1/2 of stars are found in a binary
- Binary systems can also contain compact objects

- If the two masses come close enough to each other, mass transfer (MT)

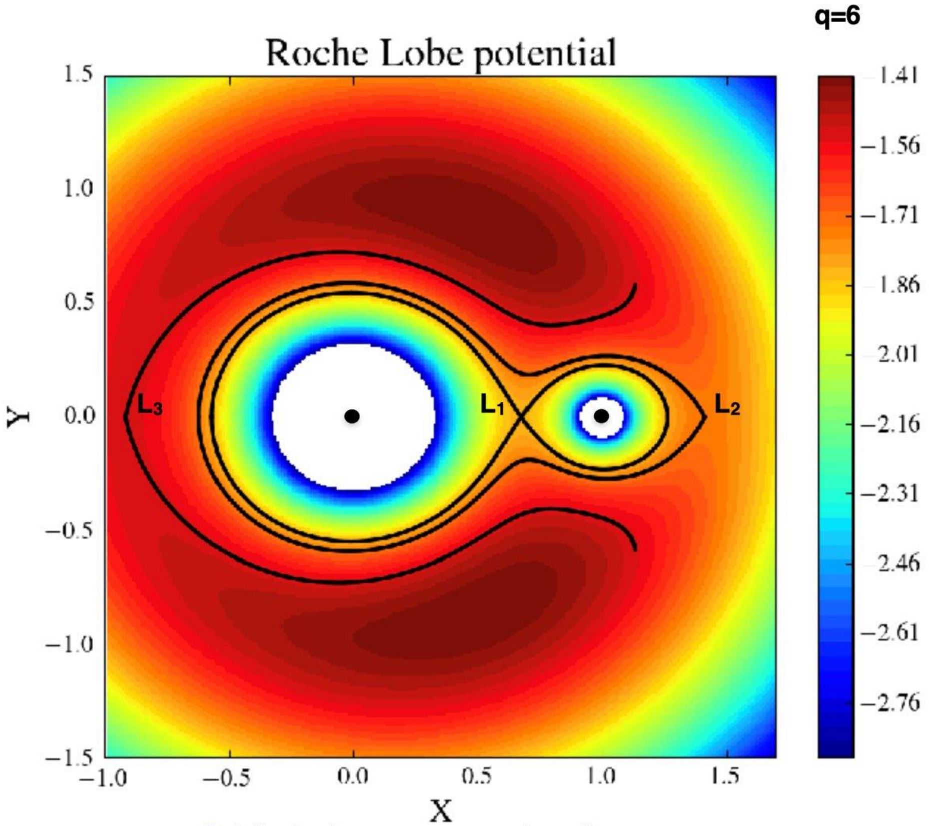
- MT crucial to:
  - X-ray binaries
  - Type Ia supernova
  - Short period compact binaries (e.g. two merging BHs)



Roche geometry

- Use a CS co-rotating with the two masses, origin at COM
- Find the Roche potential: $$\Phi_{\rm Roche}(\vec{r}) = -\frac{Gm_1}{|\vec{r}-\vec{r}_1|} - \frac{Gm_2}{|\vec{r}-\vec{r}_2|} - \frac{1}{2}\left(\vec{\omega}_{\rm K} \times \vec{r}\right)^2 ,$$
  - Critical equipotential defines Roche lobe (RL)
  - Material w/in each RL bound to star
- For each RL, define equiv. radius for sphere:

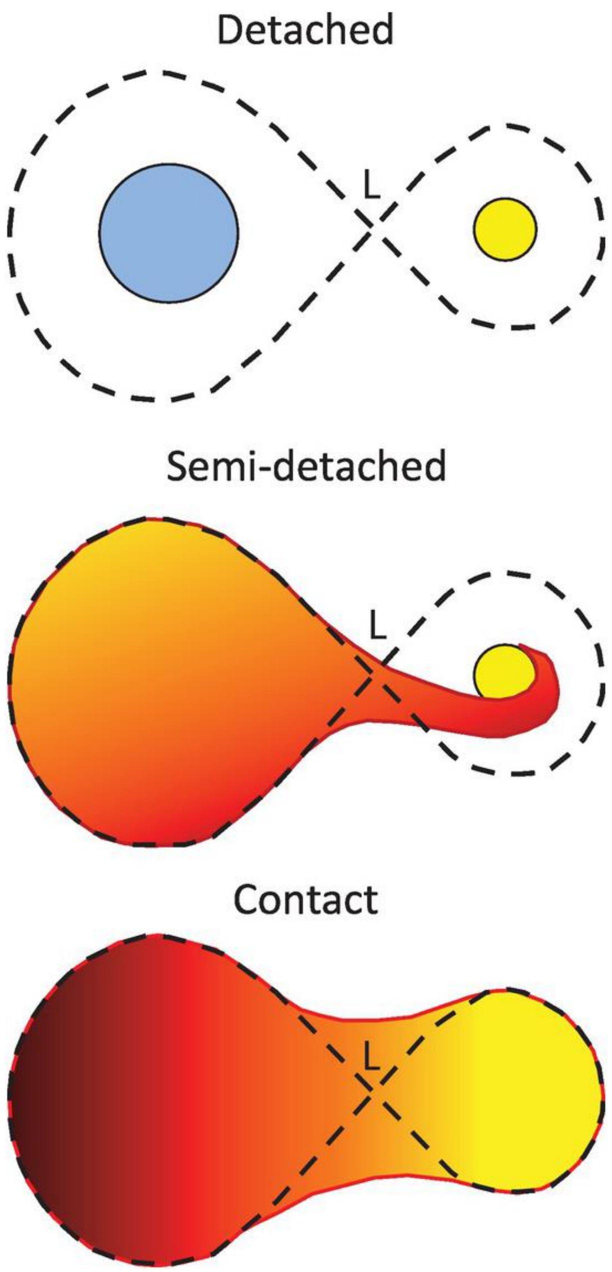
$$\frac{R_{\rm L,1}}{a} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1+q^{1/3})} \qquad \frac{R_{\rm L,1}}{a} \approx 0.44\frac{q^{0.33}}{(1+q)^{0.2}} \qquad q = M_1/M_2$$



Orbital plane, corotation frame

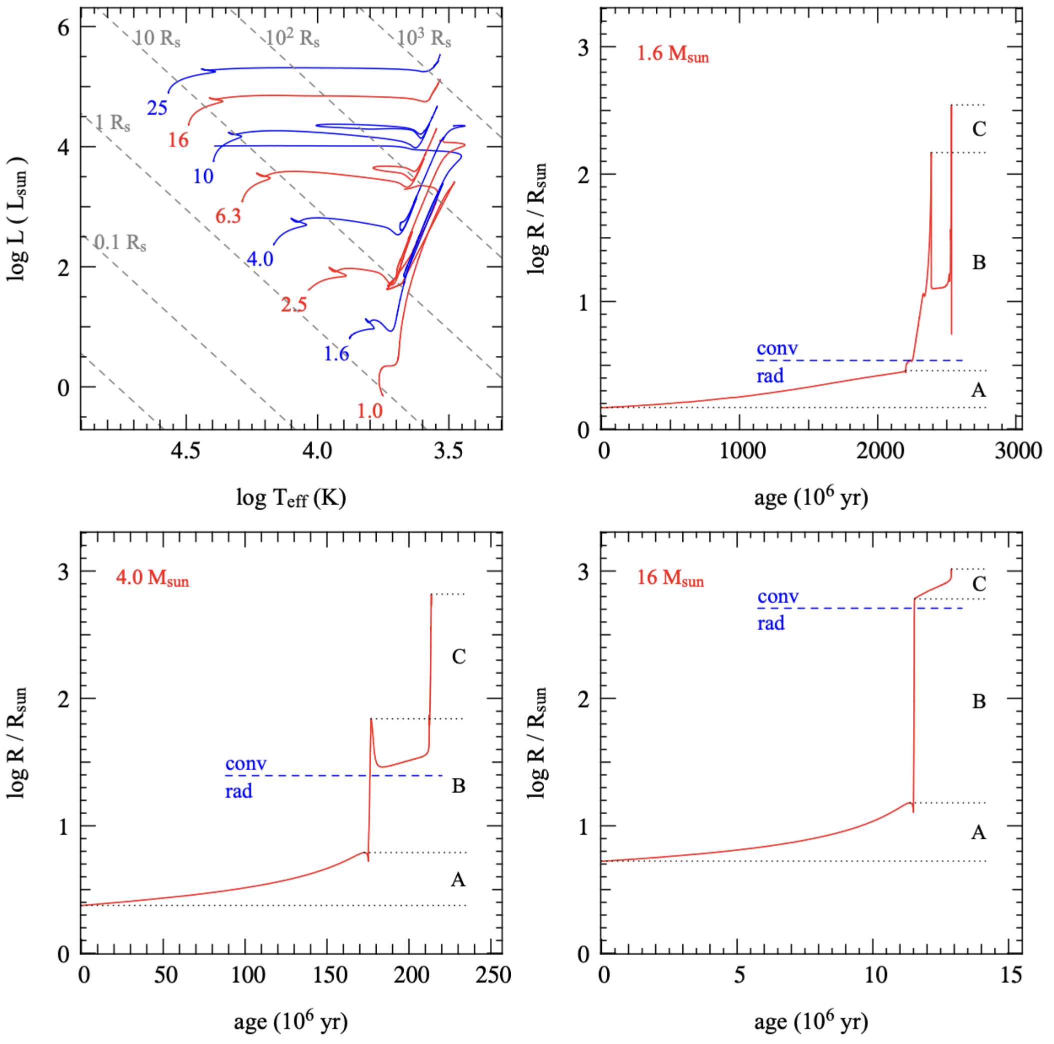
Roche lobe configurations

- 1: Detached binary
  - Both stars in their RLs
- 2: Semi-detached binary
  - One star fills its RL
  - Equilibrium not possible near L_1 ("hole" in surface) --> MT
- 3:Contact binary
  - Both stars fill their RLs
  - Shared envelope (which co-rotates w/binary)



Onset of mass transfer

- Binary system (2 stars) starts detached
- A star fills more of its RL due to:
  - Expansion of star
  - Decrease of orbital separation
- 3 cases when star overflows RL
  - Case A: on main sequence
  - Case B: expanding after H exhaustion (red giant)
  - Case C: " He exhaustion (AGB)



Conservative vs nonconservative

- Conservative: all mass lost by donor goes to companion

$$J^2 = G\frac{M_1{}^2 M_2{}^2}{M_1 + M_2} a(1 - e^2)$$

$$\dot{J} = 0 \qquad \dot{M}_\mathrm{a} = -\dot{M}_\mathrm{d}$$

$$\frac{\dot{a}}{a} = 2\left(\frac{M_\mathrm{d}}{M_\mathrm{a}} - 1\right)\frac{\dot{M}_\mathrm{d}}{M_\mathrm{d}}$$

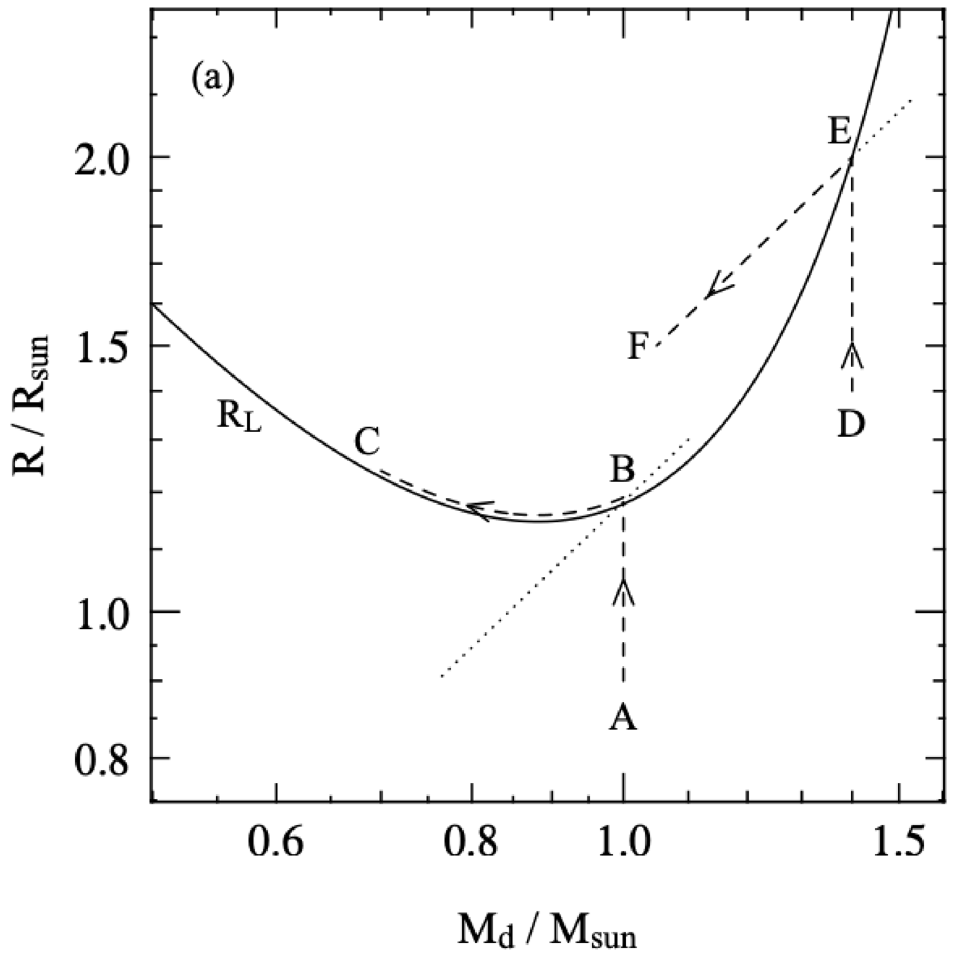
- Nonconservative: mass loss from binary
  - Wind from donor
  - Ejected from accretor (wind/jet)

$$\dot{M}_\mathrm{a} = -\beta\dot{M}_\mathrm{d}$$

(In)Stability of mass transfer

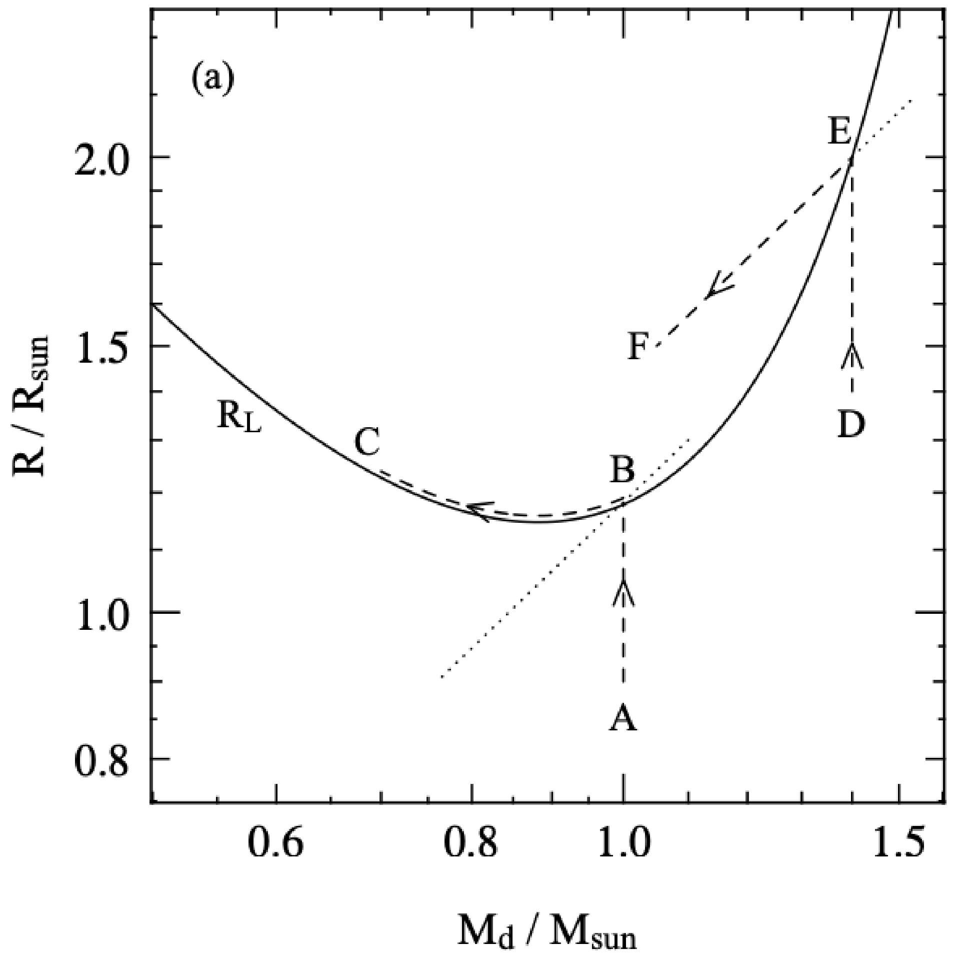
- Start: $R_d=R_L$
  - Both R_L and R_d will change due to MT
- Instability: $R_d > R_L$
  - More mass transfer, more expansion...
  - $$\zeta_* \equiv \frac{dlogR_d}{dlogM} < \frac{dlogR_L}{dlogM} \equiv \zeta_L$$
  - Power law index if $R_d \propto M^{\zeta_*}$ $R_L \propto M^{\zeta_L}$
  - Graphically:
    - R_L response to ML "steeper" than radius response
  - Mathematically:
    - $R_d - R_L > 0$, but $R_d = R_L$ initially, so $\delta R_d > \delta R_L$

$$\frac{\delta R_d}{R_d}\frac{1}{\delta logM} < \frac{\delta R_L}{R_L}\frac{1}{\delta logM}$$



Stability of mass transfer

- Start: R_d=R_L
  - Both R_L and R will change due to MT
- Stability : $R_d \leq R_L$
  - $$\zeta_* \equiv \frac{dlogR_d}{dlogM} \geq \frac{dlogR_L}{dlogM} \equiv \zeta_L$$
  - Radius response to ML "steeper" than RL response
  - Note: $\zeta_* > 0$ (< 0), star contracts (expands)
    - Same for ζ_L



What is response of star ζ_* ?

Stability:

$$\zeta_* \equiv \frac{dlogR_d}{dlogM} \geq \frac{dlogR_L}{dlogM} \equiv \zeta_L$$

- Star responds on 2 different time scales

- Dynamical
  - hydrostatic readjustment, short compared to thermal
  - Stability: $\zeta_{ad} \geq \zeta_L$

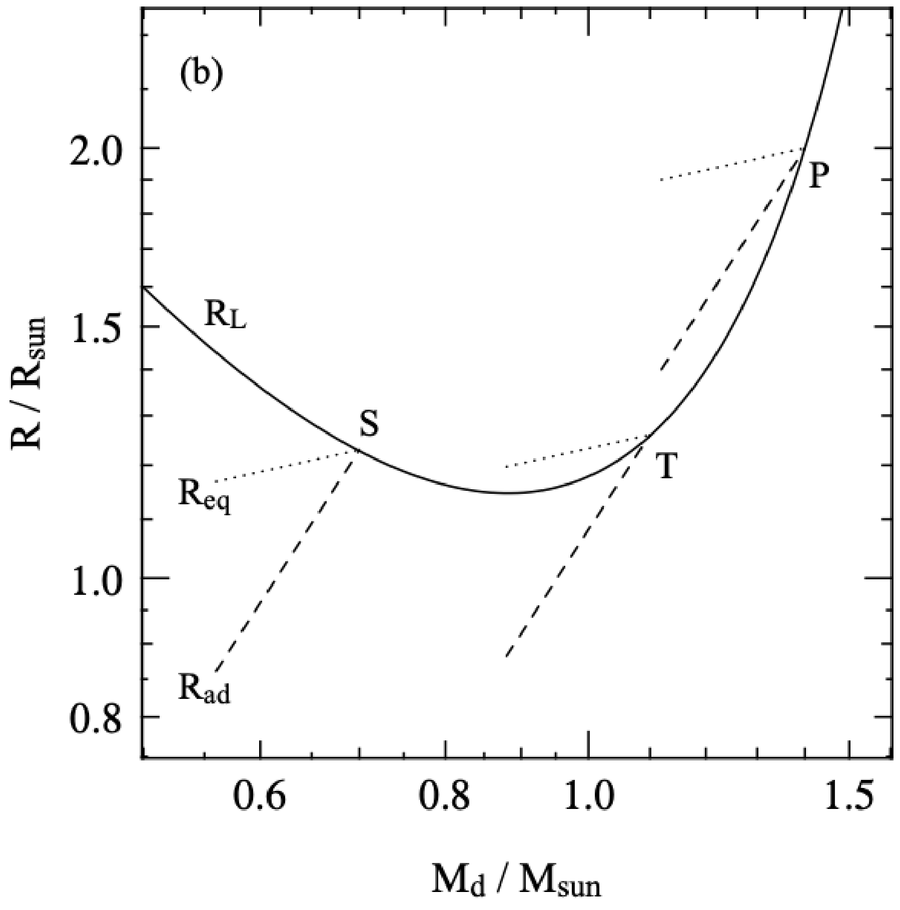
$$\zeta_{\rm ad} \equiv \left(\frac{d\log R}{d\log M}\right)_{\rm ad}$$

- Thermal
  - Only relevant if stable on dynamical timescale
  - "Thermal stability": $\zeta_{eq} \geq \zeta_L$

$$\zeta_{\rm eq} \equiv \left(\frac{d\log R}{d\log M}\right)_{\rm eq}.$$

Putting it together: 3 cases

- $\zeta_L \leq \min(\zeta_{ad}, \zeta_{eq})$
  - Secularly stable mass transfer
    - On nuclear timescales
- $\zeta_{ad} \geq \zeta_L > \zeta_{eq}$
  - Donor initially shrinks (on dynamical timescale)
  - But thermal response pushes R_d to R_L
  - Thermal-timescale mass transfer
- $\zeta_L > \zeta_{ad}$
  - Dynamically unstable



What are values of ζ_L?

- **For conservative MT**: $\zeta_{\mathrm{L}} = 2.13q - 1.67$

$$\frac{R_{\mathrm{L},1}}{a} \approx 0.44 \frac{q^{0.33}}{(1+q)^{0.2}} \qquad \frac{\dot{a}}{a} = 2\left(\frac{M_{\mathrm{d}}}{M_{\mathrm{a}}} - 1\right)\frac{\dot{M}_{\mathrm{d}}}{M_{\mathrm{d}}}$$

- Above q = 0.78, $\zeta_L > 0$
  - R_L of donor shrinks with mass loss
  - Higher mass ratio, MT more likely to be unstable

Instability:

$$\zeta_* \equiv \frac{dlogR_d}{dlogM} < \frac{dlogR_L}{dlogM} \equiv \zeta_L$$

What are values of ζ_{eq}?

$$\zeta_{eq} \equiv \left(\frac{d \log R}{d \log M}\right)_{eq}.$$

- ZAMS stars (homogenous)
  - ($M \geq 1\, M_\odot$) $\zeta_{eq} \approx 0.6$
  - ($M \lesssim 1\, M_\odot$) $\zeta_{eq} \approx 1.0$.
- Non-homogeneous: stability decreases
  - $\zeta_{eq} \lesssim 0$ for fairly evolved MS stars
  - Often assumed that $\zeta_{eq} \approx 0$ for post-MS phases
    - Detailed modeling: not quite

What are values of ζ_{ad}?

$$\zeta_{\rm ad} \equiv \left(\frac{d\log R}{d\log M}\right)_{\rm ad}$$

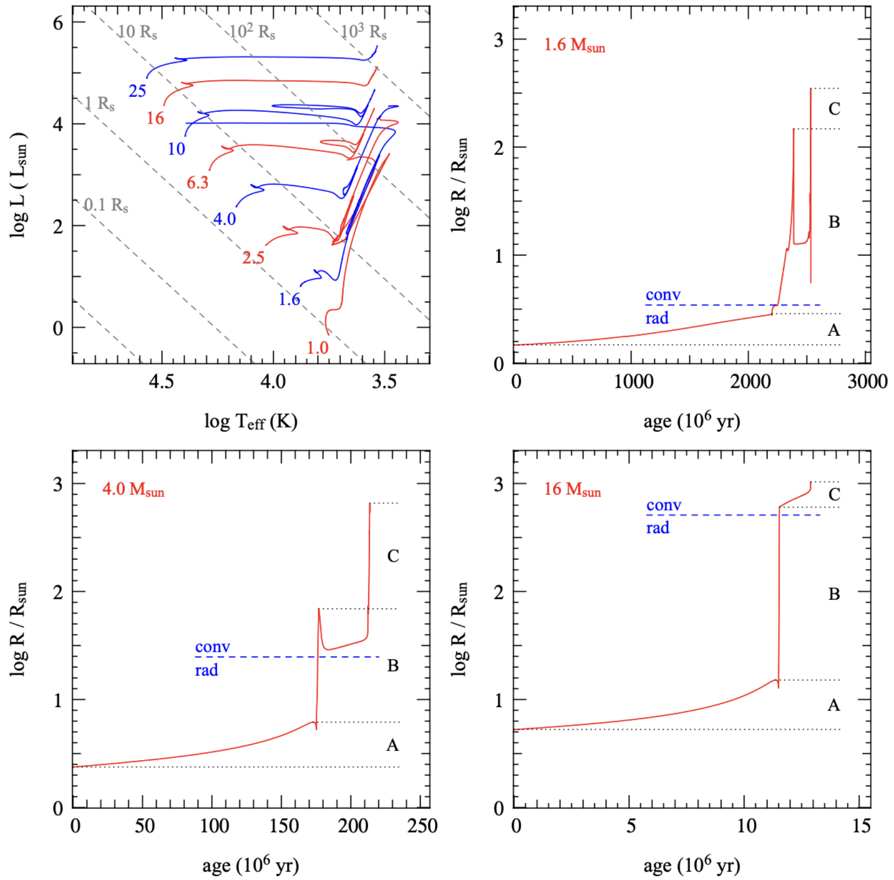
- Radiative envelope
  - Contract upon ML
  - i.e. $\zeta_{ad} \gg 0$
- Convective envelopes
  - expand or keep a roughly constant radius
  - $\zeta_{ad} \lesssim 0$
  - Pure convective: ζ_{ad} = -1/3
- For instability with convective donor:
  - $\zeta_L >$ 0 --> q > 0.78
  - For binary w/2 stars: q>1 (higher mass evolves first)
    - always unstable??

Instability:

$$\zeta_* \equiv \frac{dlogR_d}{dlogM} < \frac{dlogR_L}{dlogM} \equiv \zeta_L$$

Unstable or stable for different stages?

- Case C: always convective, unstable
- Case B: convective/unstable except for high mass donors
- Case A:
  - Unstable if $\zeta_L > \zeta_{ad} >> 0$ (radiative env)
    - Needs high q, unlikely
  - Thermal timescale MT likely:
    - $\zeta_{eq} \approx 0$
    - $\zeta_l > \zeta_{eq}$ --> q> 0.78



3 cases when star overflows RL

- Case A: on main sequence
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Stellar remnant examples

Stability:

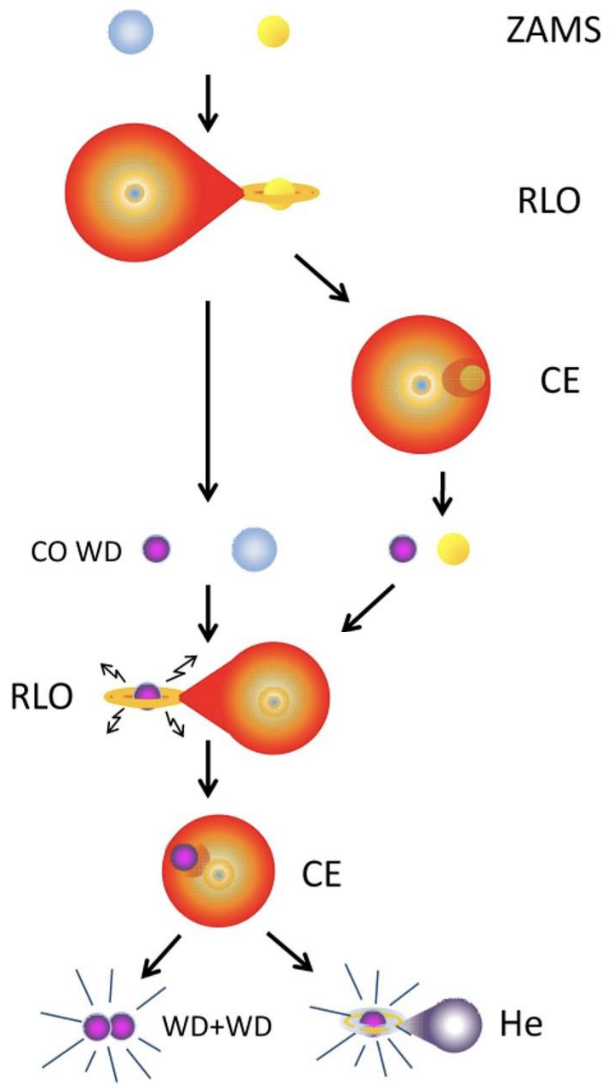
$$\zeta_* \equiv \frac{dlogR_d}{dlogM} \geq \frac{dlogR_L}{dlogM} \equiv \zeta_L$$

$$\zeta_L = 2.13q - 1.67$$

- Classical nova
  - WD accretes matter from MS companion
  - $q = M_{MS}/M_{WD} <= 1$
- Low mass X-ray binary
  - NS/BH (>1.4 $M_{\odot}$) accretes from star
  - With $q = M_{Ms}/M_{NS} <=1$
- AM CVn with WD donor
  - Less massive WD (larger) donates mass to more massive WD
  - q < 1, so can be stable (orbit widens)

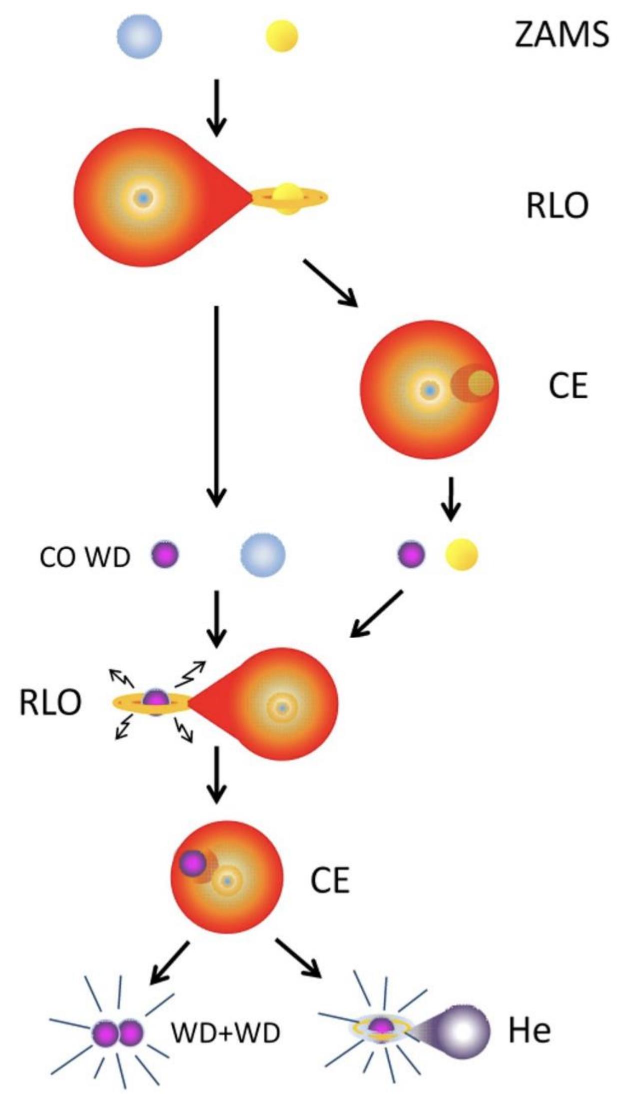
Common envelope event

- Unstable mass transfer -> CEE
- Drastic reduction in separation
  - Creates short-period binaries (double NS, BH, WDs)
- How many CE stages needed for low mass, short period DWD?
  - 2nd stage MT (after first WD formed)
    - Low mass WD + conv star
    - High mass ratio q --> CEE makes sense
  - First stage?
    - We said: convective donors + donor more massive = unstable MT ...



Common envelope event

- Parametrize CE with cons. of energy (α_{CE}) where envelope is unbound
- For *some* observed DWDs
  - The younger WD formed from MT at larger orbital separation
    - Second MT at larger orbit than first
  - If assume 2 CEEs
    - First CEE unphysical! Orbit expands rather than contracts
    - Either α_{CE} doesn't work
    - Or not unstable MT



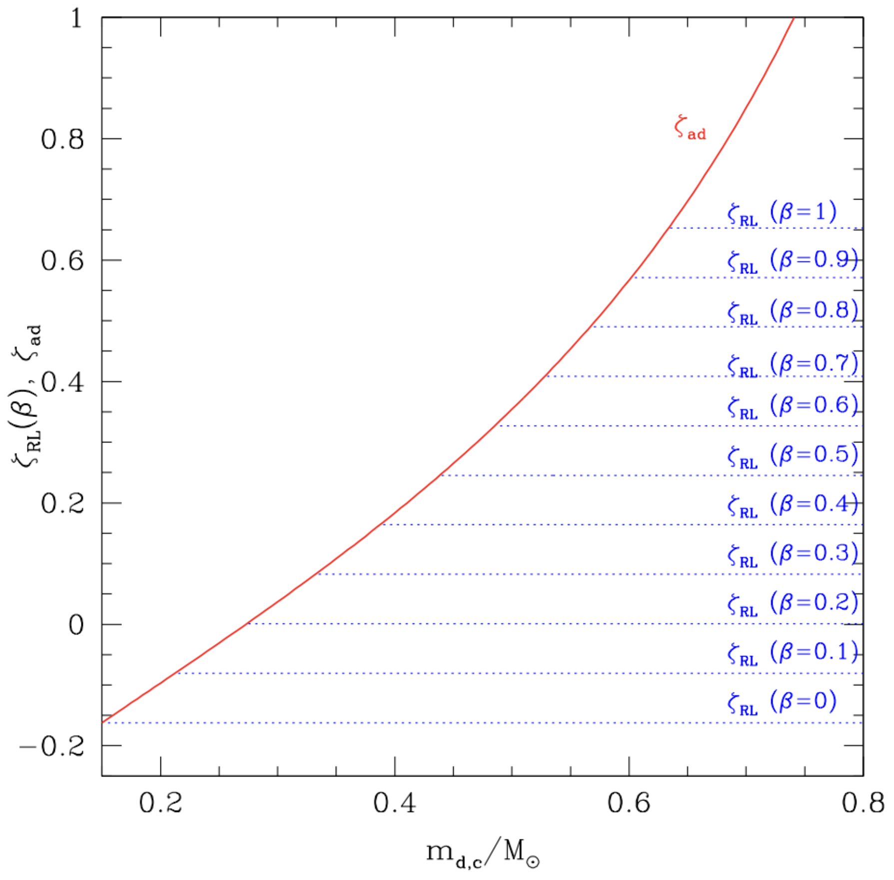
Convective donors -> always unstable MT?

- For stability, $\zeta_{ad} > \zeta_L$
- Nonconservative MT decreases ζ_L
- Finite core of RGB star increases ζ_{ad}
  - -1/3 if completely convective
- Makes stability easier

Stability:

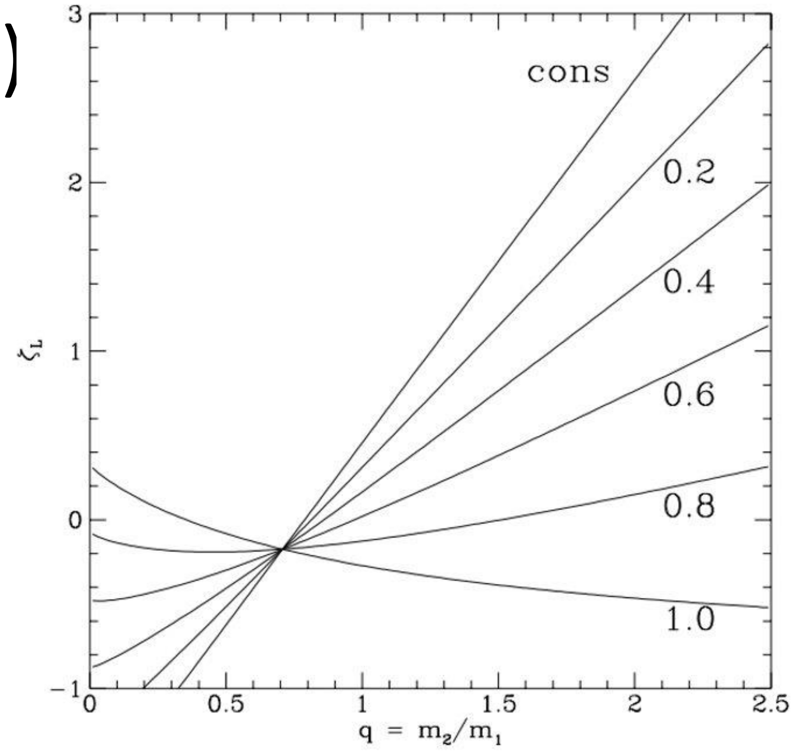
$$\zeta_* \equiv \frac{dlogR_d}{dlogM} \geq \frac{dlogR_L}{dlogM} \equiv \zeta_L$$

$$\zeta_L = 2.13q - 1.67$$



Convective donors -> always unstable MT?

- Increasing ζ_{ad} + decreasing ζ_L increases q_{crit}
  - (q such that $\zeta_L > \zeta_{ad}$)
- One estimate: q_{crit} increases to 1.3 (from 0.78)
- Therefore, ok to have first CE stage stable



Summary

- $\zeta_L \leq \min(\zeta_{ad}, \zeta_{eq})$ = stable
  - radiative donor
  - Low mass ratio q (accretor mass > donor)
- $\zeta_L > \zeta_{ad}$ = unstable
  - ζ_L increases with mass ratio q
    - However, is decreased with nonconservative MT
  - $\zeta_{ad} <0$ for convective donor
- Convective evolved stars with $q > 1$ are best for unstable MT
  - But exact q important for predicting CEE