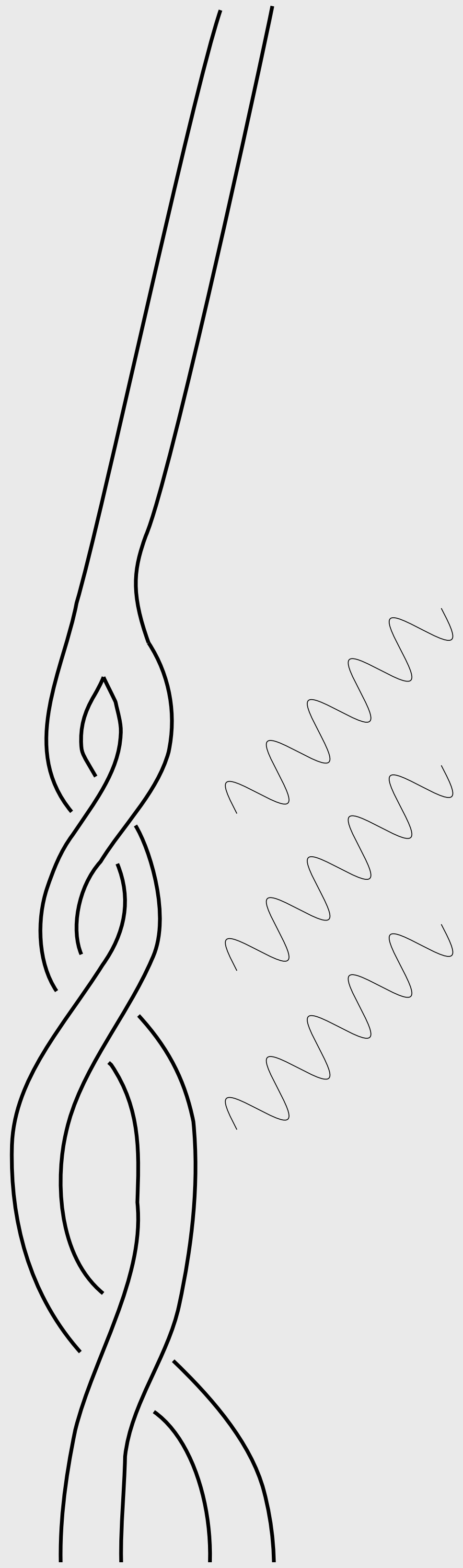
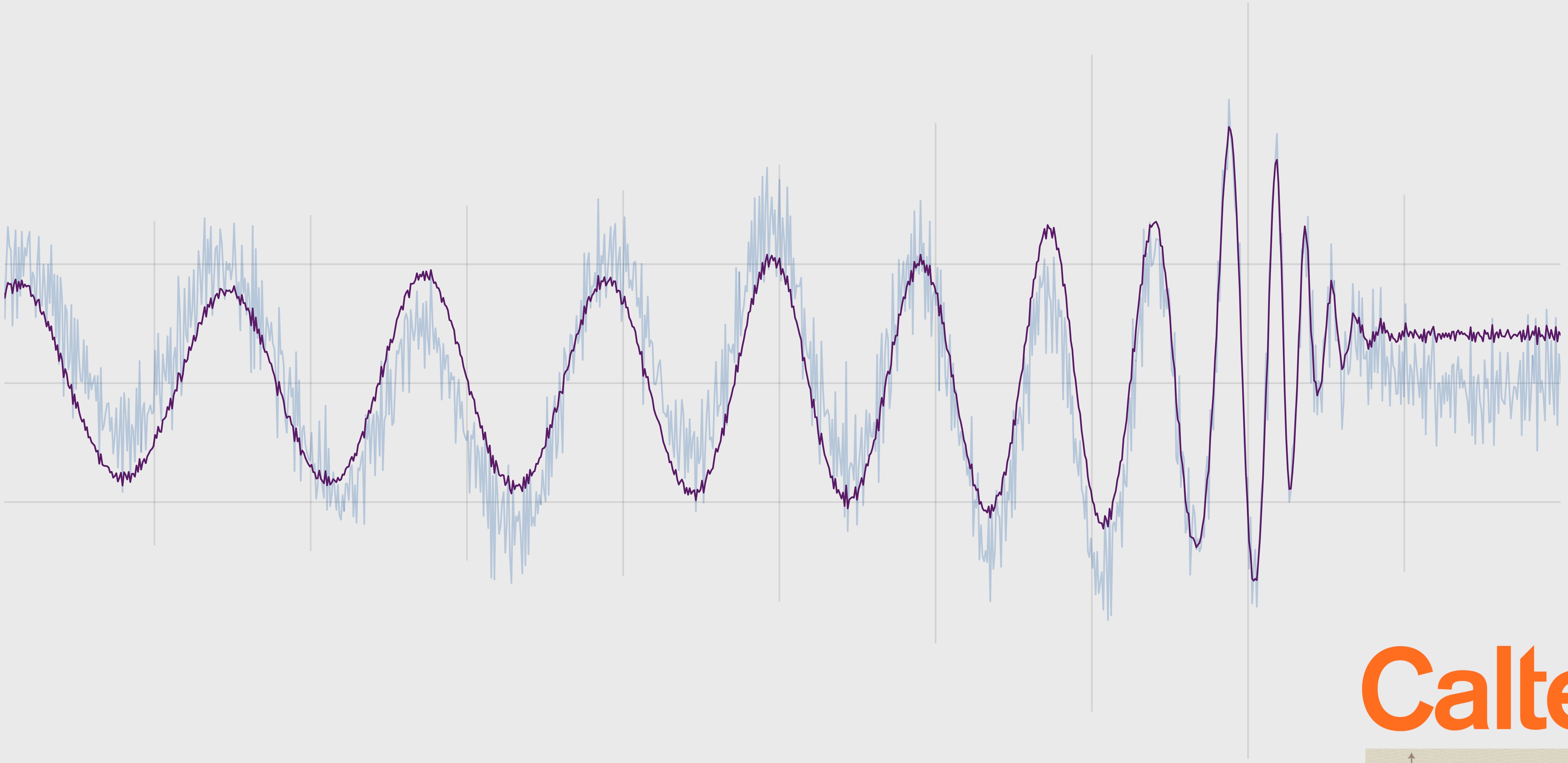
What Gravitational Waves Actually Look Like





Keefe Mitman
ARC Seminar, October 24th

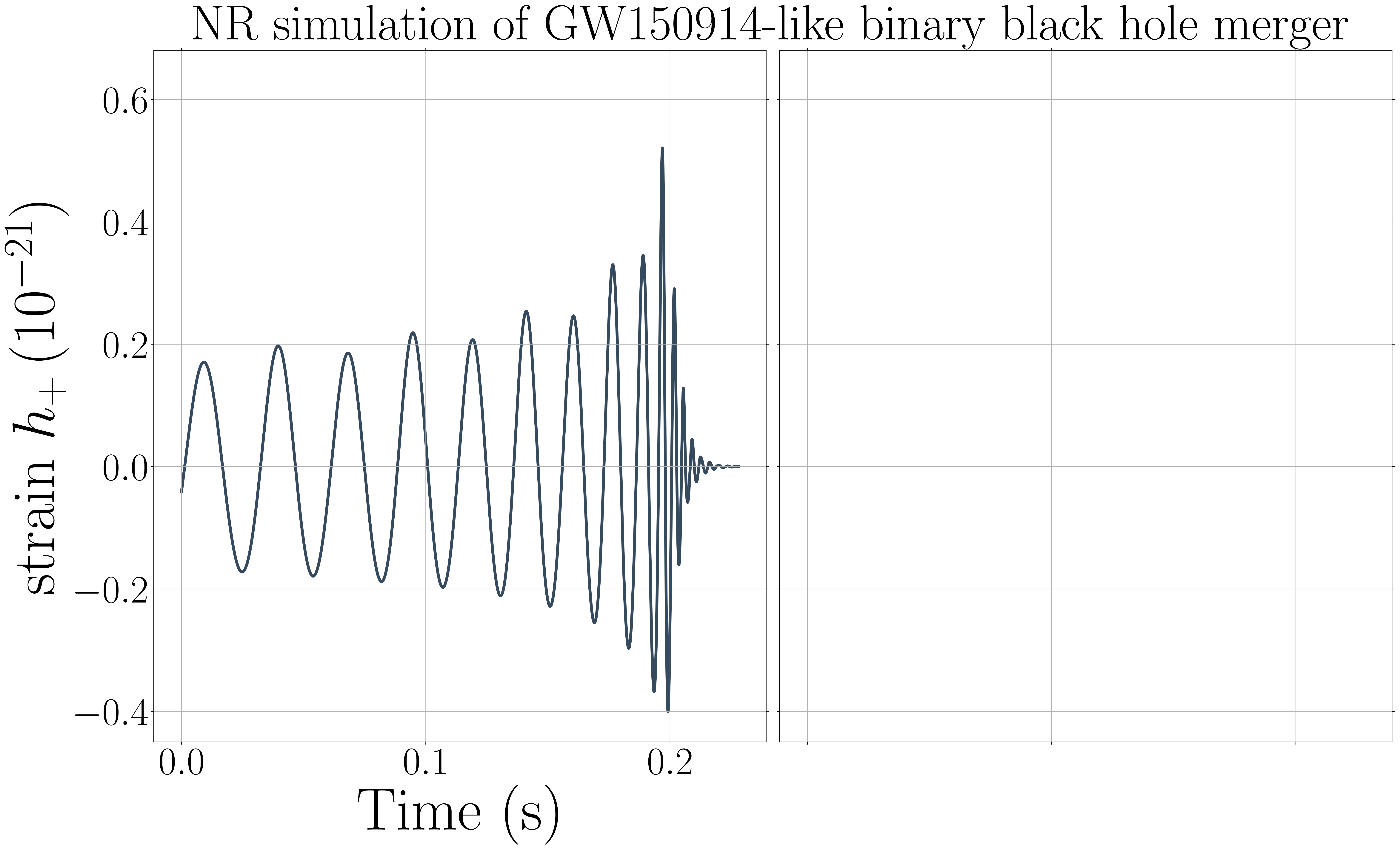
Caltech

Simulating eXtreme Spacetimes
Black holes, neutron stars, and beyond...

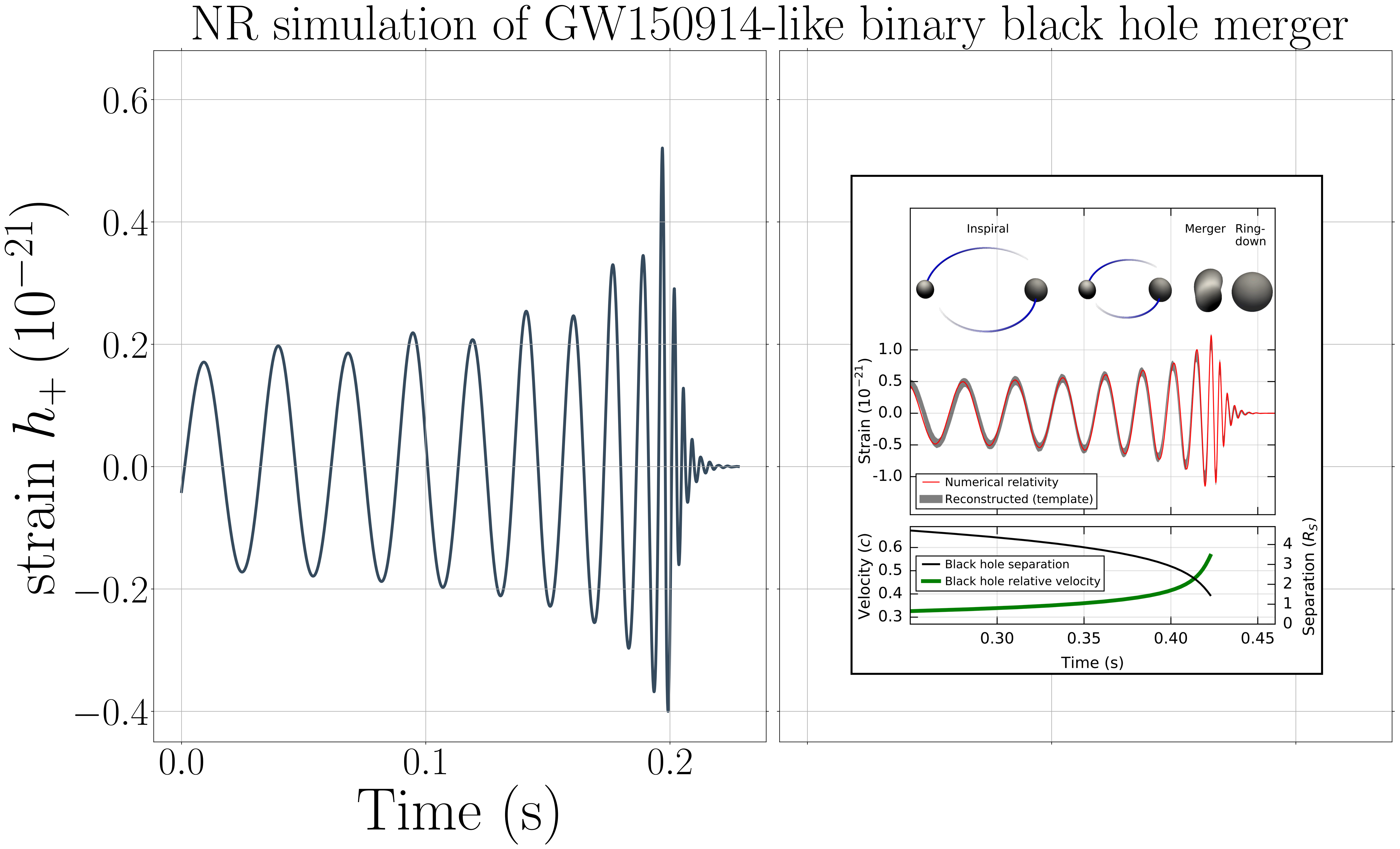
Outline

- **Numerical Lies**
  - You mean we've had it wrong this whole time?
- **Figuring out the Truth for Ourselves**
  - Why is it obvious that we've been wrong? Is it really that simple?
- **Gravitational Memory**
  - Derivation from BMS charges and fluxes
  - Categorizing the types of memory
- **Gravitational Memory in NR Simulations**
  - Why we didn't see it at first: Extrapolation vs. CCE
  - Looking toward the future

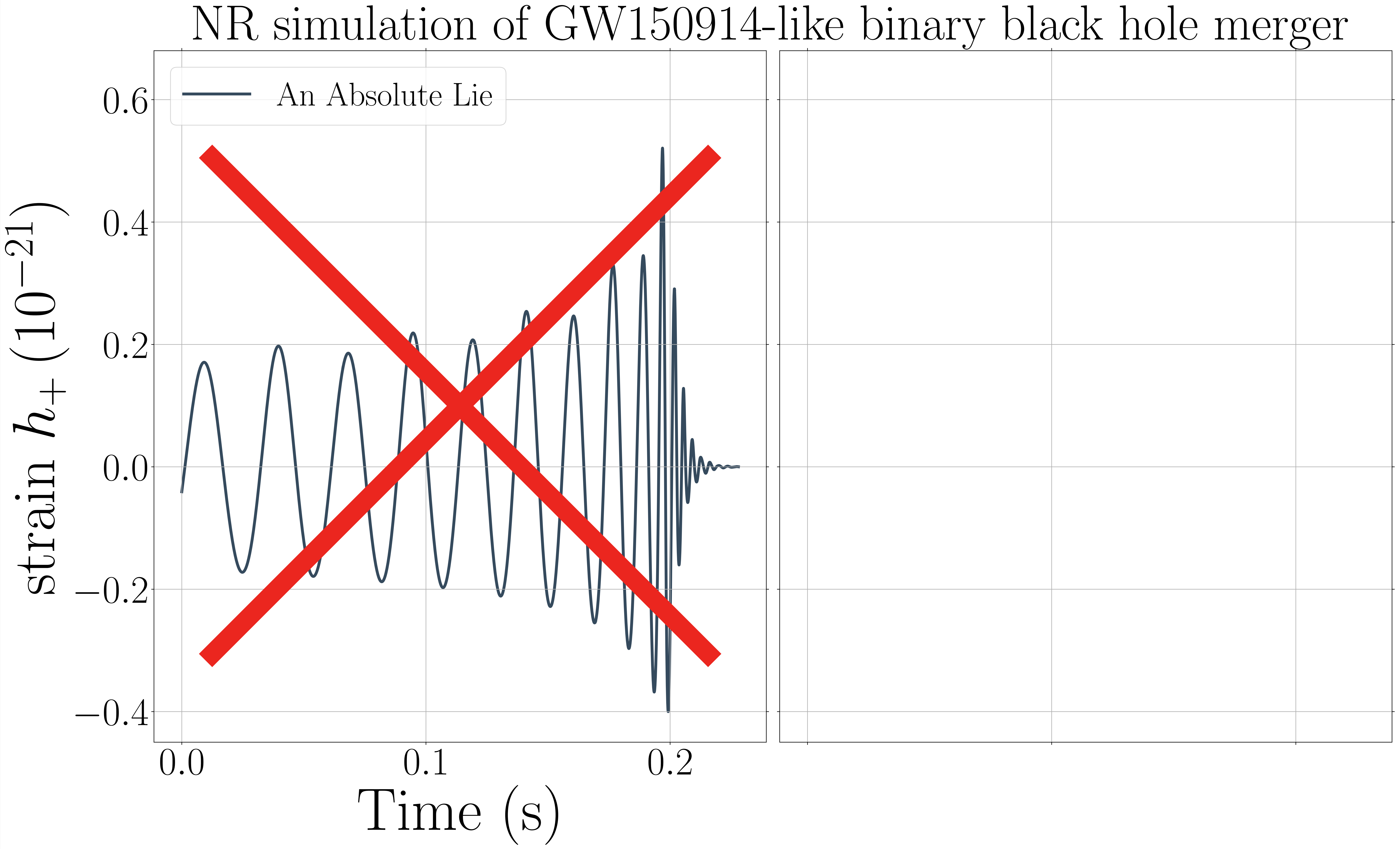
Wow! Our First Detection!



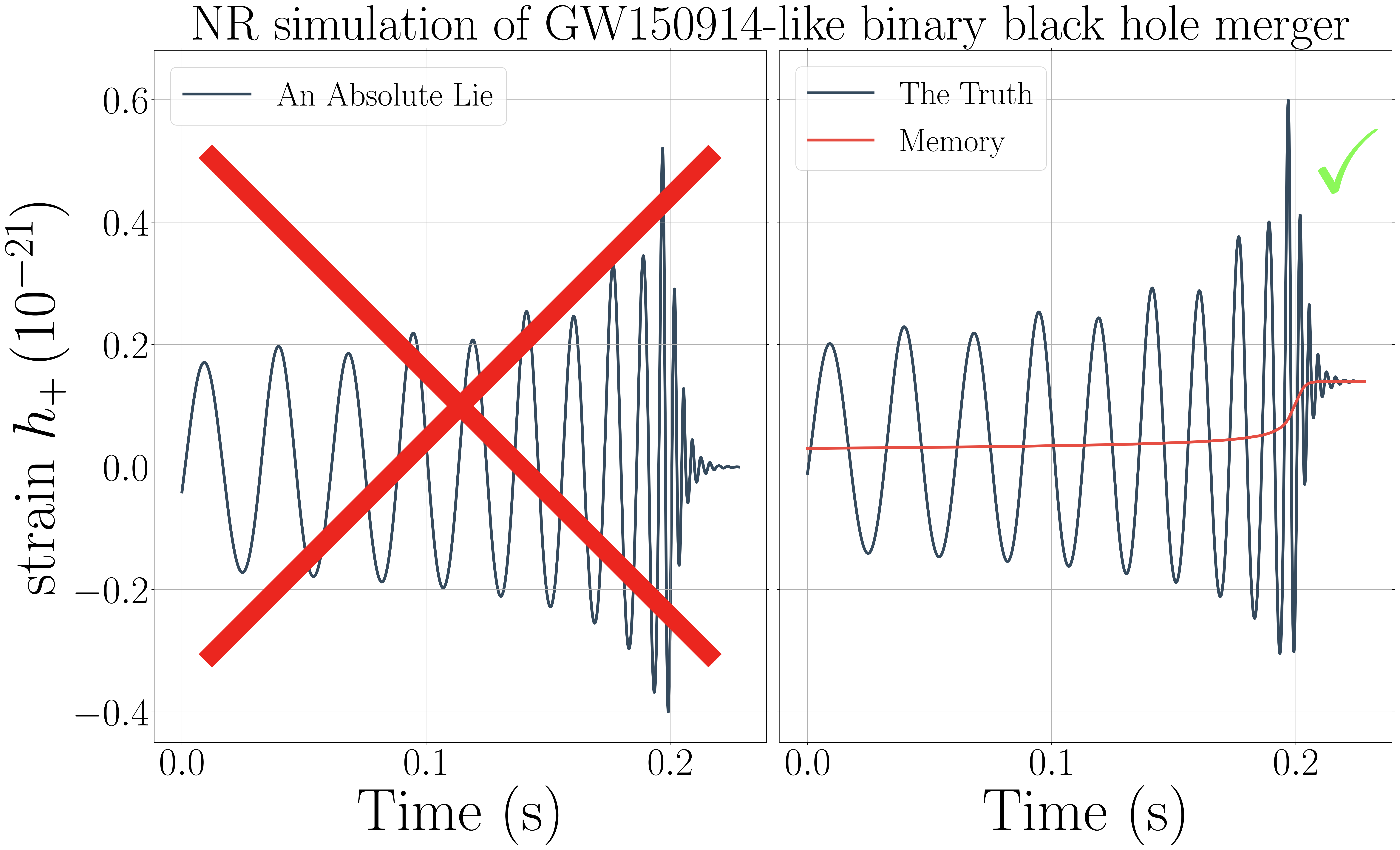
Wow! Our First Detection!



~~Wow! Our First Detection!~~ It's a Lie!



~~Wow! Our First Detection!~~ The Truth!



Gravitational wave emitted by a compact binary system

The Problem:

- Consider two point masses m_1 and m_2 which are orbiting one another in circular orbits of radius r_1 and r_2 in the xy-plane about the center of mass with angular velocity ω. Using the quadrupole formula, which comes from first-order perturbation theory, compute the gravitational wave (GW) strain emitted by the binary system.

Gravitational wave emitted by a compact binary system

The Problem:

- Consider two point masses m_1 and m_2 which are orbiting one another in circular orbits of radius r_1 and r_2 in the xy-plane about the center of mass with angular velocity ω. Using the quadrupole formula, which comes from **first-order perturbation theory**, compute the gravitational wave (GW) strain emitted by the binary system.

Gravitational wave emitted by a compact binary system

The Solution:

- Let $G = c = 1$. According to the. quadrupole formula we have $\bar{h}_{ij}(t,r) = \frac{2}{r}\ddot{I}_{ij}(t-r)$, where $\bar{h}_{ij}$ is the trace-reversed metric perturbation and $I_{ij} = \int \rho(\vec{r})\left(r_i r_j - \frac{1}{3}||\vec{r}||^2\delta_{ij}\right) d^3\vec{r}$ is the quadrupole moment tensor with mass density $\rho(\vec{r}) = \frac{1}{r}\sum_{i=1}^{2} m_i \delta(r-r_i)\delta(\phi_i)\delta(z)$.

Gravitational wave emitted by a compact binary system

The Solution:

- Let $G = c = 1$. According to the. quadrupole formula we have $\bar{h}_{ij}(t, r) = \frac{2}{r}\ddot{I}_{ij}(t - r)$, where $\bar{h}_{ij}$ is the trace-reversed metric perturbation and $I_{ij} = \int \rho(\vec{r})\left(r_i r_j - \frac{1}{3}||\vec{r}||^2\delta_{ij}\right) d^3\vec{r}$ is the quadrupole moment tensor with mass density $\rho(\vec{r}) = \frac{1}{r}\sum_{i=1}^{2} m_i\delta(r - r_i)\delta(\phi_i)\delta(z)$.
- Compute the integral, convert to TT, and use the quadrupole formula to obtain

$$h_+ = \frac{4}{r}M\nu x\left[\frac{1}{2}\left(1 + \cos^2(\theta)\right)\cos\left(2\left(\omega t - \phi\right)\right)\right]$$

$$h_\times = \frac{4}{r}M\nu x\left[\cos(\theta)\sin\left(2\left(\omega t - \phi\right)\right)\right]$$

with $M \equiv m_1 + m_2$, $\nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$, and $x \equiv (M\omega)^{\frac{2}{3}}$ is the post-Newtonian parameter.

Gravitational wave emitted by a compact binary system

The Solution:

- Let $G = c = 1$. According to the. quadrupole formula we have $\bar{h}_{ij}(t,r) = \frac{2}{r}\ddot{I}_{ij}(t-r)$, where $\bar{h}_{ij}$ is the trace-reversed metric perturbation and $I_{ij} = \int \rho(\vec{r})\left(r_i r_j - \frac{1}{3}||\vec{r}||^2\delta_{ij}\right) d^3\vec{r}$ is the quadrupole moment tensor with mass density $\rho(\vec{r}) = \frac{1}{r}\sum_{i=1}^{2} m_i\delta(r-r_i)\delta(\phi_i)\delta(z)$.
- In terms of spin-weighted harmonics, this is

$$h = \sum_{\ell=0}\sum_{|m|\leq\ell} h_{\ell m}\, {}_{-2}Y_{\ell m}(\theta,\phi)$$

where

$$h^{\ell m} = \frac{2}{r}M\nu x\sqrt{\frac{16\pi}{5}}\mathscr{H}^{(\ell,m)}e^{-im\omega t}$$

with

$$\mathscr{H}^{(2,+2)} = 1$$

$$\mathscr{H}^{(2,-2)} = 1$$

Gravitational wave emitted by a compact binary system

The Solution:

- Let $G = c = 1$. According to the. quadrupole formula we have $\bar{h}_{ij}(t, r) = \frac{2}{r}\ddot{I}_{ij}(t - r)$, where $\bar{h}_{ij}$ is the trace-reversed metric perturbation and $I_{ij} = \int \rho(\vec{r})\left(r_i r_j - \frac{1}{3}||\vec{r}||^2 \delta_{ij}\right) d^3\vec{r}$ is the quadrupole moment tensor with mass density $\rho(\vec{r}) = \frac{1}{r}\sum_{i=1}^{2} m_i \delta(r - r_i)\delta(\phi_i)\delta(z)$.
- In terms of spin-weighted harmonics, this is

$$h = \sum_{\ell=0} \sum_{|m|\leq\ell} h_{\ell m\ -2}Y_{\ell m}(\theta, \phi)$$

where

$$h^{\ell m} = \frac{2}{r}M\nu x\sqrt{\frac{16\pi}{5}}\mathscr{H}^{(\ell,m)}e^{-im\omega t}$$

with

$$\mathscr{H}^{(2,+2)} = 1$$

$$\mathscr{H}^{(2,-2)} = 1$$

- This is the 0th order PN result!

Gravitational wave emitted by a compact binary system

The Solution:

- Let $G = c = 1$. According to the. quadrupole formula we have $\bar{h}_{ij}(t,r) = \frac{2}{r}\ddot{I}_{ij}(t-r)$, where $\bar{h}_{ij}$ is the trace-reversed metric perturbation and $I_{ij} = \int \rho(\vec{r})\left(r_i r_j - \frac{1}{3}||\vec{r}||^2\delta_{ij}\right) d^3\vec{r}$ is the quadrupole moment tensor with mass density $\rho(\vec{r}) = \frac{1}{r}\sum_{i=1}^{2} m_i \delta(r-r_i)\delta(\phi_i)\delta(z)$.
- In terms of spin-weighted harmonics, this is

$$h = \sum_{\ell=0}\sum_{|m|\leq\ell} h_{\ell m}\, {}_{-2}Y_{\ell m}(\theta,\phi)$$

where

$$h^{\ell m} = \frac{2}{r}M\nu x\sqrt{\frac{16\pi}{5}}\mathscr{H}^{(\ell,m)}e^{-im\omega t}$$

with

$$\mathscr{H}^{(2,+2)} = 1$$

$$\mathscr{H}^{(2,-2)} = 1$$

- This is (almost) the 0th order PN result!

Gravitational wave emitted by a compact binary system

Accounting for Energy Loss:

- Can complete the 0th order PN by accounting for the energy carried by the emitted GWs

Gravitational wave emitted by a compact binary system

Accounting for Energy Loss:

- Can complete the 0th order PN by accounting for the energy carried by the emitted GWs

$$\frac{dE}{dt} = -\mathscr{F}$$

where

$$E = -\frac{1}{2}\frac{M^2\nu}{r}$$

$$\mathscr{F} = \frac{1}{5}\langle \dddot{I}_{ij}\dddot{I}_{ij}\rangle = \frac{32}{5}M^2\nu^2 r^4\omega^6$$

Gravitational wave emitted by a compact binary system

Accounting for Energy Loss:

- Can complete the 0th order PN by accounting for the energy carried by the emitted GWs
  $$\frac{dE}{dt} = -\mathcal{F}$$
  where
  $$E = -\frac{1}{2}\frac{M^2\nu}{r}$$
  $$\mathcal{F} = \frac{1}{5}\langle \dddot{I}_{ij}\dddot{I}_{ij}\rangle = \frac{32}{5}M^2\nu^2r^4\omega^6$$
- This, with $\omega^2 = M/r^3$, yields the ODE
  $$\dot{\omega} = \frac{96}{5}\nu(M\omega)^{\frac{5}{3}}\omega^2$$

Gravitational wave emitted by a compact binary system

Accounting for Energy Loss:

- Can complete the 0th order PN by accounting for the energy carried by the emitted GWs
$$\frac{dE}{dt} = -\mathcal{F}$$
where
$$E = -\frac{1}{2}\frac{M^2\nu}{r}$$
$$\mathcal{F} = \frac{1}{5}\langle \dddot{I}_{ij}\dddot{I}_{ij}\rangle = \frac{32}{5}M^2\nu^2r^4\omega^6$$
- This, with $\omega^2 = M/r^3$, yields the ODE
$$\dot{\omega} = \frac{96}{5}\nu(M\omega)^{\frac{5}{3}}\omega^2$$
- By integrating this ODE, we find that the phase of system evolves as
$$\psi(x) \equiv \int \omega\, dt = \frac{x^{-\frac{5}{2}}}{32\nu}$$

Gravitational wave emitted by a compact binary system

Accounting for Energy Loss:

- Can complete the 0th order PN by accounting for the energy carried by the emitted GWs
$$\frac{dE}{dt} = -\mathcal{F}$$
where
$$E = -\frac{1}{2}\frac{M^2\nu}{r}$$
$$\mathcal{F} = \frac{1}{5}\langle \dddot{I}_{ij}\dddot{I}_{ij}\rangle = \frac{32}{5}M^2\nu^2r^4\omega^6$$
- This, with $\omega^2 = M/r^3$, yields the ODE
$$\dot{\omega} = \frac{96}{5}\nu(M\omega)^{\frac{5}{3}}\omega^2$$
- By integrating this ODE, we find that the phase of system evolves as
$$\psi(x) \equiv \int \omega\, dt = \frac{x^{-\frac{5}{2}}}{32\nu}$$
- Our strain is then
$$h^{\ell m} = \frac{2}{r}M\nu x\sqrt{\frac{16\pi}{5}}\mathcal{H}^{(\ell,m)}e^{-im\psi(x)}$$

Gravitational wave emitted by a compact binary system

Accounting for Energy Loss:

- Can complete the 0th order PN by accounting for the energy carried by the emitted GWs
$$\frac{dE}{dt} = -\mathscr{F}$$
where
$$E = -\frac{1}{2}\frac{M^2\nu}{r}$$
$$\mathscr{F} = \frac{1}{5}\langle \dddot{I}_{ij}\dddot{I}_{ij}\rangle = \frac{32}{5}M^2\nu^2 r^4\omega^6$$
- This, with $\omega^2 = M/r^3$, yields the ODE
$$\dot{\omega} = \frac{96}{5}\nu(M\omega)^{\frac{5}{3}}\omega^2$$
- By integrating this ODE, we find that the phase of system evolves as
$$\psi(x) \equiv \int \omega\, dt = \frac{x^{-\frac{5}{2}}}{32\nu}$$
- Our strain is then
$$h^{\ell m} = \frac{2}{r}M\nu x\sqrt{\frac{16\pi}{5}}\mathscr{H}^{(\ell,m)}e^{-im\psi(x)}$$
- Last, note that this ODE also provides the time-dependence of x via
$$\dot{x} = \frac{64}{5}\frac{\nu}{M}x^5$$

Gravitational wave emitted by a compact binary system

Accounting for Energy Loss:

- Can complete the 0th order PN by accounting for the energy carried by the emitted GWs
$$\frac{dE}{dt} = -\mathcal{F}$$
where
$$E = -\frac{1}{2}\frac{M^2\nu}{r}$$
$$\mathcal{F} = \frac{1}{5}\langle \dddot{I}_{ij}\dddot{I}_{ij}\rangle = \frac{32}{5}M^2\nu^2 r^4\omega^6$$
- This, with $\omega^2 = M/r^3$, yields the ODE
$$\dot{\omega} = \frac{96}{5}\nu(M\omega)^{\frac{5}{3}}\omega^2$$
- By integrating this ODE, we find that the phase of system evolves as
$$\psi(x) \equiv \int \omega\, dt = \frac{x^{-\frac{5}{2}}}{32\nu}$$
- Our strain is then
$$h^{\ell m} = \frac{2}{r}M\nu x\sqrt{\frac{16\pi}{5}}\mathcal{H}^{(\ell,m)}e^{-im\psi(x)}$$
- Last, note that this ODE also provides the time-dependence of x via
$$\dot{x} = \frac{64}{5}\frac{\nu}{M}x^5$$

All first-order perturbation theory!

Gravitational wave emitted by a compact binary system

Considering Nonlinearities:

- A useful tensor besides the Riemann tensor is the Weyl tensor $C_{\mu\nu\rho\sigma}$ because it is more directly related to the strain

Gravitational wave emitted by a compact binary system

Considering Nonlinearities:

- A useful tensor besides the Riemann tensor is the Weyl tensor $C_{\mu\nu\rho\sigma}$ because it is more directly related to the strain
- It too obeys Bianchi identities, usually written in terms of the Weyl scalars:

$$\psi_4 = -\ddot{\bar{\sigma}}$$

$$\psi_3 = -\eth\dot{\bar{\sigma}}$$

$$\mathrm{Im}\left[\psi_2\right] = -\mathrm{Im}\left[\eth^2\bar{\sigma} + \sigma\dot{\bar{\sigma}}\right]$$

$$\dot{\psi}_2 = \eth\psi_3 + \sigma\psi_4$$

$$\dot{\psi}_1 = \eth\psi_2 + 2\sigma\psi_3$$

$$\dot{\psi}_0 = \eth\psi_1 + 3\sigma\psi_2$$

Gravitational wave emitted by a compact binary system

Considering Nonlinearities:

- A useful tensor besides the Riemann tensor is the Weyl tensor $C_{\mu\nu\rho\sigma}$ because it is more directly related to the strain
- It too obeys Bianchi identities, usually written in terms of the Weyl scalars:

$$\psi_4 = -\ddot{\bar{\sigma}}$$

$$\psi_3 = -\eth\dot{\bar{\sigma}}$$

$$\text{Im}\left[\psi_2\right] = -\text{Im}\left[\eth^2\bar{\sigma} + \sigma\dot{\bar{\sigma}}\right]$$

$$\dot{\psi}_2 = \eth\psi_3 + \sigma\psi_4$$

$$\dot{\psi}_1 = \eth\psi_2 + 2\sigma\psi_3$$

$$\dot{\psi}_0 = \eth\psi_1 + 3\sigma\psi_2$$

Gravitational wave emitted by a compact binary system

Considering Nonlinearities:

- A useful tensor besides the Riemann tensor is the Weyl tensor $C_{\mu\nu\rho\sigma}$ because it is more directly related to the strain
- It too obeys Bianchi identities, usually written in terms of the Weyl scalars:

$$\psi_4 = -\ddot{\bar{\sigma}}$$

$$\psi_3 = -\eth\dot{\bar{\sigma}}$$

$$\mathrm{Im}\left[\psi_2\right] = -\mathrm{Im}\left[\eth^2\bar{\sigma} + \sigma\dot{\bar{\sigma}}\right]$$

$$\dot{\psi}_2 = \eth\psi_3 + \sigma\psi_4$$

$$\dot{\psi}_1 = \eth\psi_2 + 2\sigma\psi_3$$

$$\dot{\psi}_0 = \eth\psi_1 + 3\sigma\psi_2$$

- If we rearrange this and integrate we have

$$\eth^2\bar{\sigma} = -\left(\psi_2 + \sigma\dot{\bar{\sigma}}\right) + \int_{-\infty}^{u} |\dot{\sigma}|^2 du$$

Gravitational wave emitted by a compact binary system

Considering Nonlinearities:

- A useful tensor besides the Riemann tensor is the Weyl tensor $C_{\mu\nu\rho\sigma}$ because it is more directly related to the strain
- It too obeys Bianchi identities, usually written in terms of the Weyl scalars:

$$\psi_4 = -\ddot{\bar{\sigma}}$$

$$\psi_3 = -\eth\dot{\bar{\sigma}}$$

$$\mathrm{Im}\left[\psi_2\right] = -\,\mathrm{Im}\left[\eth^2\bar{\sigma} + \sigma\dot{\bar{\sigma}}\right]$$

$$\dot{\psi}_2 = \eth\psi_3 + \sigma\psi_4$$

$$\dot{\psi}_1 = \eth\psi_2 + 2\sigma\psi_3$$

$$\dot{\psi}_0 = \eth\psi_1 + 3\sigma\psi_2$$

- If we rearrange this and integrate we have

$$\eth^2\bar{\sigma} = -\left(\psi_2 + \sigma\dot{\bar{\sigma}}\right) + \int_{-\infty}^{u} |\dot{\sigma}|^2 du$$

- ψ_2 is effectively what we just computed

Gravitational wave emitted by a compact binary system

Considering Nonlinearities:

- A useful tensor besides the Riemann tensor is the Weyl tensor $C_{\mu\nu\rho\sigma}$ because it is more directly related to the strain
- It too obeys Bianchi identities, usually written in terms of the Weyl scalars:

$$\psi_4 = -\ddot{\bar{\sigma}}$$
$$\psi_3 = -\eth\dot{\bar{\sigma}}$$
$$\mathrm{Im}\left[\psi_2\right] = -\mathrm{Im}\left[\eth^2\bar{\sigma} + \sigma\dot{\bar{\sigma}}\right]$$
$$\dot{\psi}_2 = \eth\psi_3 + \sigma\psi_4$$
$$\dot{\psi}_1 = \eth\psi_2 + 2\sigma\psi_3$$
$$\dot{\psi}_0 = \eth\psi_1 + 3\sigma\psi_2$$

- If we rearrange this and integrate we have

$$\eth^2\bar{\sigma} = -\left(\psi_2 + \sigma\dot{\bar{\sigma}}\right) + \int_{-\infty}^{u} |\dot{\sigma}|^2 du$$

- ψ_2 is effectively what we just computed
- $\sigma\dot{\bar{\sigma}}$ only contributes at higher PN order

Gravitational wave emitted by a compact binary system

Considering Nonlinearities:

- A useful tensor besides the Riemann tensor is the Weyl tensor $C_{\mu\nu\rho\sigma}$ because it is more directly related to the strain
- It too obeys Bianchi identities, usually written in terms of the Weyl scalars:

$$\psi_4 = -\ddot{\bar{\sigma}}$$
$$\psi_3 = -\eth\dot{\bar{\sigma}}$$
$$\mathrm{Im}\left[\psi_2\right] = -\mathrm{Im}\left[\eth^2\bar{\sigma} + \sigma\dot{\bar{\sigma}}\right]$$
$$\dot{\psi}_2 = \eth\psi_3 + \sigma\psi_4$$
$$\dot{\psi}_1 = \eth\psi_2 + 2\sigma\psi_3$$
$$\dot{\psi}_0 = \eth\psi_1 + 3\sigma\psi_2$$

- If we rearrange this and integrate we have

$$\eth^2\bar{\sigma} = -\left(\psi_2 + \sigma\dot{\bar{\sigma}}\right) + \int_{-\infty}^{u} |\dot{\sigma}|^2 du$$

- ψ_2 is effectively what we just computed
- $\sigma\dot{\bar{\sigma}}$ only contributes at higher PN order
- but $\int_{-\infty}^{u} |\dot{\sigma}|^2 du$ is… interesting…

Gravitational wave emitted by a compact binary system

Finding a Missing PN Term:

- This energy-flux term can be written as

$$\mathscr{E}^{\ell m} \equiv \left(\int_{-\infty}^{u} |\dot{\sigma}|^2 du \right)^{\ell m}$$

$$= \frac{1}{4} \sum_{\ell_1, |m_1| \leq \ell_1} \sum_{\ell_2, |m_2| \leq \ell_2} \left[\int_{S^2} \overline{{}_0Y_{\ell m} {}_{-2}Y_{\ell_1 m_1}} {}_{-2}Y_{\ell m} d\Omega \right] \left[\int \overline{\dot{h}^{\ell_1 m_1}} \dot{h}^{\ell_2 m_2} du \right]$$

Gravitational wave emitted by a compact binary system

Finding a Missing PN Term:

- This energy-flux term can be written as

$$\mathcal{E}^{\ell m} \equiv \left(\int_{-\infty}^{u} |\dot{\sigma}|^2 du \right)^{\ell m}$$

$$= \frac{1}{4} \sum_{\ell_1, |m_1| \leq \ell_1} \sum_{\ell_2, |m_2| \leq \ell_2}$$

$$\left[\int_{S^2} \overline{{}_0Y_{\ell m} {}_{-2}Y_{\ell_1 m_1}} {}_{-2}Y_{\ell m} d\Omega \right]$$

$$\left[\int \overline{\dot{h}^{\ell_1 m_1}} \dot{h}^{\ell_2 m_2} du \right]$$

Easy to compute using Wigner-D matrices (requires $m = -m_1 + m_2$)

Gravitational wave emitted by a compact binary system

Finding a Missing PN Term:

- This energy-flux term can be written as

$$\mathscr{E}^{\ell m} \equiv \left(\int_{-\infty}^{u} |\dot{\sigma}|^2 du \right)^{\ell m}$$

$$= \frac{1}{4} \sum_{\ell_1,|m_1|\leq\ell_1} \sum_{\ell_2,|m_2|\leq\ell_2}$$

$$\left[\int_{S^2} \overline{{}_0Y_{\ell m}} \, \overline{{}_{-2}Y_{\ell_1 m_1}} \, {}_{-2}Y_{\ell m} d\Omega \right]$$

$$\left[\int \overline{\dot{h}^{\ell_1 m_1}} \dot{h}^{\ell_2 m_2} du \right]$$

Easy to compute using Wigner-D matrices (requires $m = -m_1 + m_2$)

Change to an integral over x and use $\psi(x)$ and $\dot{x}$ from before

Gravitational wave emitted by a compact binary system

Finding a Missing PN Term:

- This energy-flux term can be written as

$$\mathcal{E}^{\ell m} \equiv \left(\int_{-\infty}^{u} |\dot{\sigma}|^2 du \right)^{\ell m}$$

$$= \frac{1}{4} \sum_{\ell_1,|m_1|\leq\ell_1} \sum_{\ell_2,|m_2|\leq\ell_2}$$

$$\left[\int_{S^2} \overline{{}_0Y_{\ell m}\,{}_{-2}Y_{\ell_1 m_1}}\,{}_{-2}Y_{\ell m} d\Omega \right]$$

$$\left[\int \overline{\dot{h}^{\ell_1 m_1}} \dot{h}^{\ell_2 m_2} du \right]$$

Easy to compute using Wigner-D matrices (requires $m = -m_1 + m_2$)

Change to an integral over x and use $\psi(x)$ and $\dot{x}$ from before

$$\int \dot{x}\, \overline{h'_{\ell_1 m_1}} h'_{\ell_2 m_2} dx$$

Gravitational wave emitted by a compact binary system

Finding a Missing PN Term:

- This energy-flux term can be written as

$$\mathscr{E}^{\ell m} \equiv \left(\int_{-\infty}^{u} |\dot{\sigma}|^2 du \right)^{\ell m}$$

$$= \frac{1}{4} \sum_{\ell_1, |m_1| \leq \ell_1} \sum_{\ell_2, |m_2| \leq \ell_2}$$

$$\left[\int_{S^2} \overline{{}_0Y_{\ell m}\, {}_{-2}Y_{\ell_1 m_1}}\, {}_{-2}Y_{\ell m} d\Omega \right]$$

$$\left[\int \overline{\dot{h}^{\ell_1 m_1}} \dot{h}^{\ell_2 m_2} du \right]$$

Easy to compute using Wigner-D matrices (requires $m = -m_1 + m_2$)

Change to an integral over x and use $\psi(x)$ and $\dot{x}$ from before

$$\int \dot{x}\, \overline{h'_{\ell_1 m_1}} h'_{\ell_2 m_2} dx$$

$$\sim \int \dot{x}\, x^2 \left(\psi'(x)\right)^2 dx$$

Gravitational wave emitted by a compact binary system

Finding a Missing PN Term:

- This energy-flux term can be written as

$$\mathscr{E}^{\ell m} \equiv \left(\int_{-\infty}^{u} |\dot{\sigma}|^2 du \right)^{\ell m}$$

$$= \frac{1}{4} \sum_{\ell_1,|m_1|\leq\ell_1} \sum_{\ell_2,|m_2|\leq\ell_2}$$

$$\left[\int_{S^2} \overline{{}_0Y_{\ell m} {}_{-2}Y_{\ell_1 m_1}} {}_{-2}Y_{\ell m} d\Omega \right]$$

$$\left[\int \overline{\dot{h}^{\ell_1 m_1}} \dot{h}^{\ell_2 m_2} du \right]$$

Easy to compute using Wigner-D matrices (requires $m = -m_1 + m_2$)

Change to an integral over x and use $\psi(x)$ and $\dot{x}$ from before

$$\int \dot{x}\, \overline{h'_{\ell_1 m_1}} h'_{\ell_2 m_2} dx$$

$$\sim \int \dot{x}\, x^2 \left(\psi'(x)\right)^2 dx$$

$$\psi(x) = \frac{x^{-\frac{5}{2}}}{32\nu}, \dot{x} = \frac{64}{5}\frac{\nu}{M}x^5$$

Gravitational wave emitted by a compact binary system

Finding a Missing PN Term:

- This energy-flux term can be written as

$$\mathscr{E}^{\ell m} \equiv \left(\int_{-\infty}^{u} |\dot{\sigma}|^2 du \right)^{\ell m}$$

$$= \frac{1}{4} \sum_{\ell_1, |m_1| \leq \ell_1} \sum_{\ell_2, |m_2| \leq \ell_2}$$

$$\left[\int_{S^2} \overline{{}_0Y_{\ell m}} \, \overline{{}_{-2}Y_{\ell_1 m_1}} \, {}_{-2}Y_{\ell m} d\Omega \right]$$

$$\left[\int \overline{\dot{h}^{\ell_1 m_1}} \dot{h}^{\ell_2 m_2} du \right]$$

- Carrying out these integrals yields a single spherical harmonic mode at 0th PN order:

Gravitational wave emitted by a compact binary system

Finding a Missing PN Term:

- This energy-flux term can be written as

$$\mathcal{E}^{\ell m} \equiv \left(\int_{-\infty}^{u} |\dot{\sigma}|^2 du \right)^{\ell m}$$
$$= \frac{1}{4} \sum_{\ell_1, |m_1| \leq \ell_1} \sum_{\ell_2, |m_2| \leq \ell_2}$$
$$\left[\int_{S^2} \overline{{}_0Y_{\ell m}\, {}_{-2}Y_{\ell_1 m_1}}\, {}_{-2}Y_{\ell m} d\Omega \right]$$
$$\left[\int \overline{\dot{h}^{\ell_1 m_1}} \dot{h}^{\ell_2 m_2} du \right]$$

- Carrying out these integrals yields a single spherical harmonic mode at 0th PN order:

$$h^{20} = \frac{2}{r} M\nu x \sqrt{\frac{16\pi}{5}} \frac{5}{14\sqrt{6}}$$

Gravitational wave emitted by a compact binary system

Finding a Missing PN Term:

- This energy-flux term can be written as

$$\mathcal{E}^{\ell m} \equiv \left(\int_{-\infty}^{u} |\dot{\sigma}|^2 du\right)^{\ell m}$$

$$= \frac{1}{4} \sum_{\ell_1, |m_1| \leq \ell_1} \sum_{\ell_2, |m_2| \leq \ell_2}$$

$$\left[\int_{S^2} \overline{{}_0Y_{\ell m}\,{}_{-2}Y_{\ell_1 m_1}}\,{}_{-2}Y_{\ell m}\, d\Omega\right]$$

$$\left[\int \overline{\dot{h}^{\ell_1 m_1}}\,\dot{h}^{\ell_2 m_2} du\right]$$

- Carrying out these integrals yields a single spherical harmonic mode at 0th PN order:

$$h^{20} = \frac{2}{r} M\nu x \sqrt{\frac{16\pi}{5}} \frac{5}{14\sqrt{6}}$$

- It's non-oscillatory and monotonic…

Gravitational wave emitted by a compact binary system

Finding a Missing PN Term:

- This energy-flux term can be written as

$$\mathcal{E}^{\ell m} \equiv \left(\int_{-\infty}^{u} |\dot{\sigma}|^2 du\right)^{\ell m}$$
$$= \frac{1}{4} \sum_{\ell_1,|m_1|\leq\ell_1} \sum_{\ell_2,|m_2|\leq\ell_2} \left[\int_{S^2} \overline{{}_0Y_{\ell m}}\, \overline{{}_{-2}Y_{\ell_1 m_1}}\, {}_{-2}Y_{\ell m} d\Omega\right] \left[\int \overline{\dot{h}^{\ell_1 m_1}} \dot{h}^{\ell_2 m_2} du\right]$$

- Carrying out these integrals yields a single spherical harmonic mode at 0th PN order:

$$h^{20} = \frac{2}{r} M\nu x \sqrt{\frac{16\pi}{5}} \frac{5}{14\sqrt{6}}$$

- It's non-oscillatory and monotonic…
- It's…

Gravitational wave emitted by a compact binary system

Finding a Missing PN Term:

- This energy-flux term can be written as

$$\mathcal{E}^{\ell m} \equiv \left(\int_{-\infty}^{u} |\dot{\sigma}|^2 du \right)^{\ell m}$$

$$= \frac{1}{4} \sum_{\ell_1, |m_1| \leq \ell_1} \sum_{\ell_2, |m_2| \leq \ell_2}$$

$$\left[\int_{S^2} \overline{{}_0Y_{\ell m} {}_{-2}Y_{\ell_1 m_1}} {}_{-2}Y_{\ell m} d\Omega \right]$$

$$\left[\int \overline{\dot{h}^{\ell_1 m_1}} \dot{h}^{\ell_2 m_2} du \right]$$

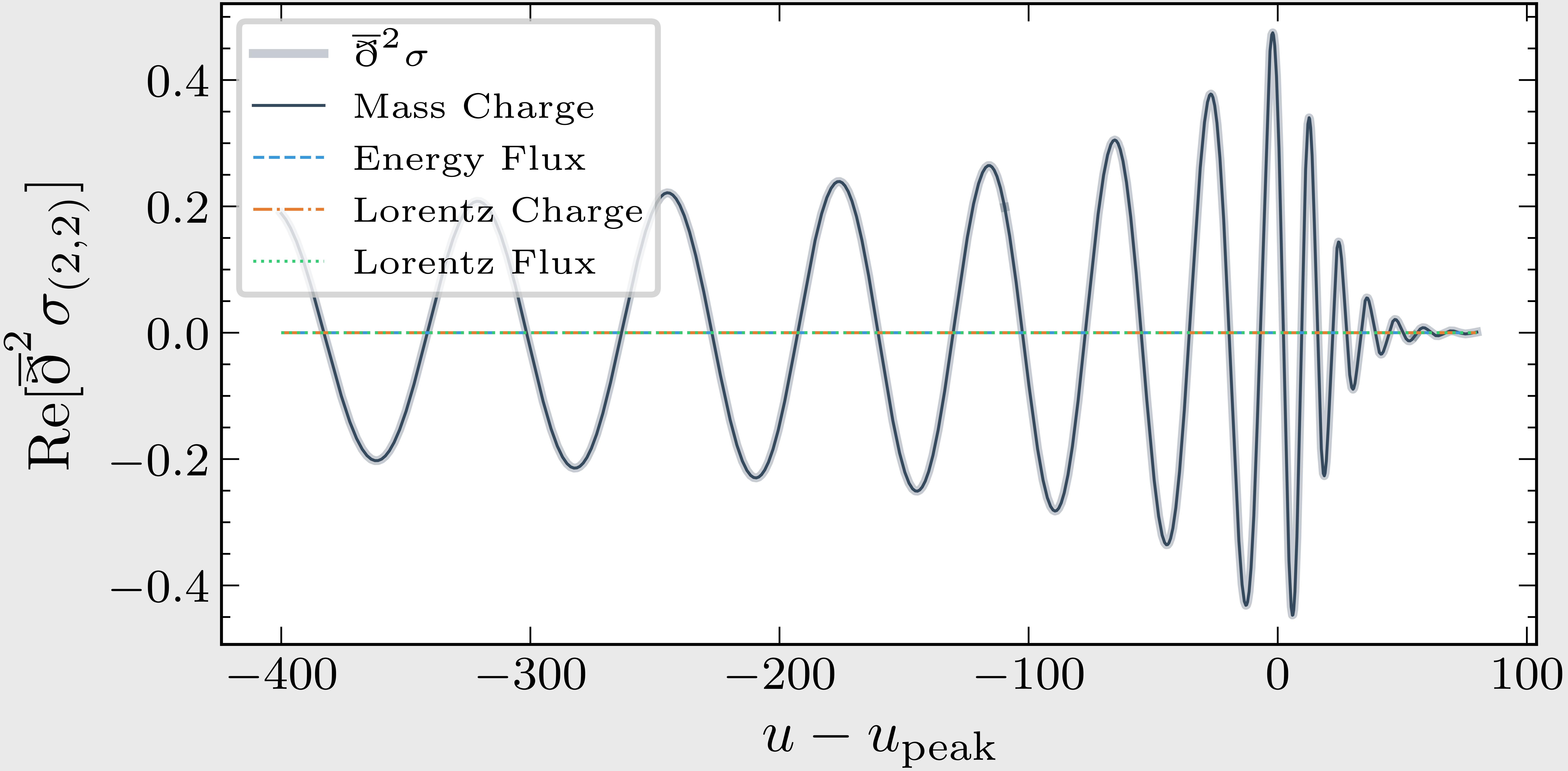
- Carrying out these integrals yields a single spherical harmonic mode at 0th PN order:

$$h^{20} = \frac{2}{r} M \nu x \sqrt{\frac{16\pi}{5}} \frac{5}{14\sqrt{6}}$$

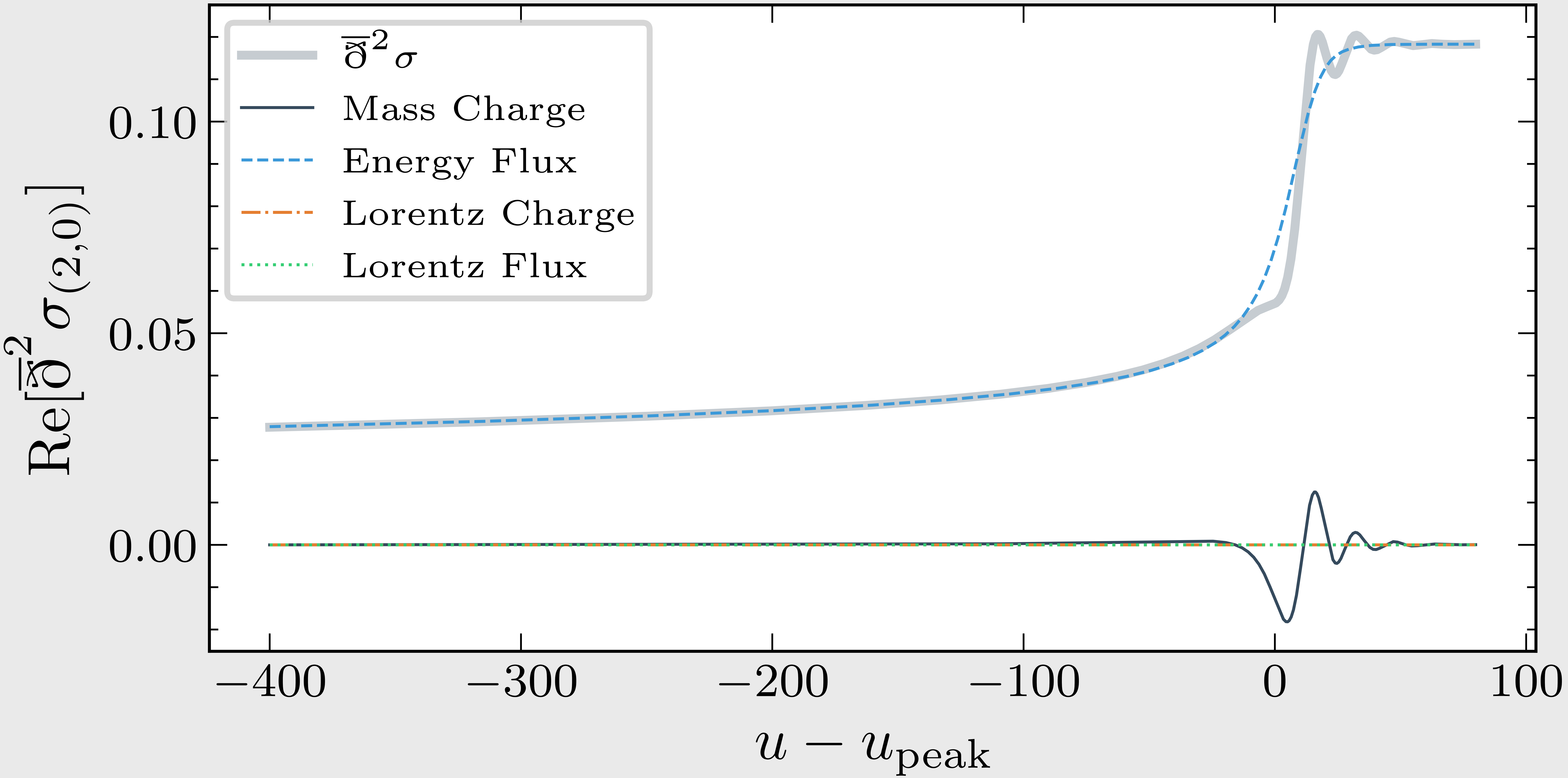
- It's non-oscillatory and monotonic…

- It's… **GRAVITATIONAL MEMORY!**

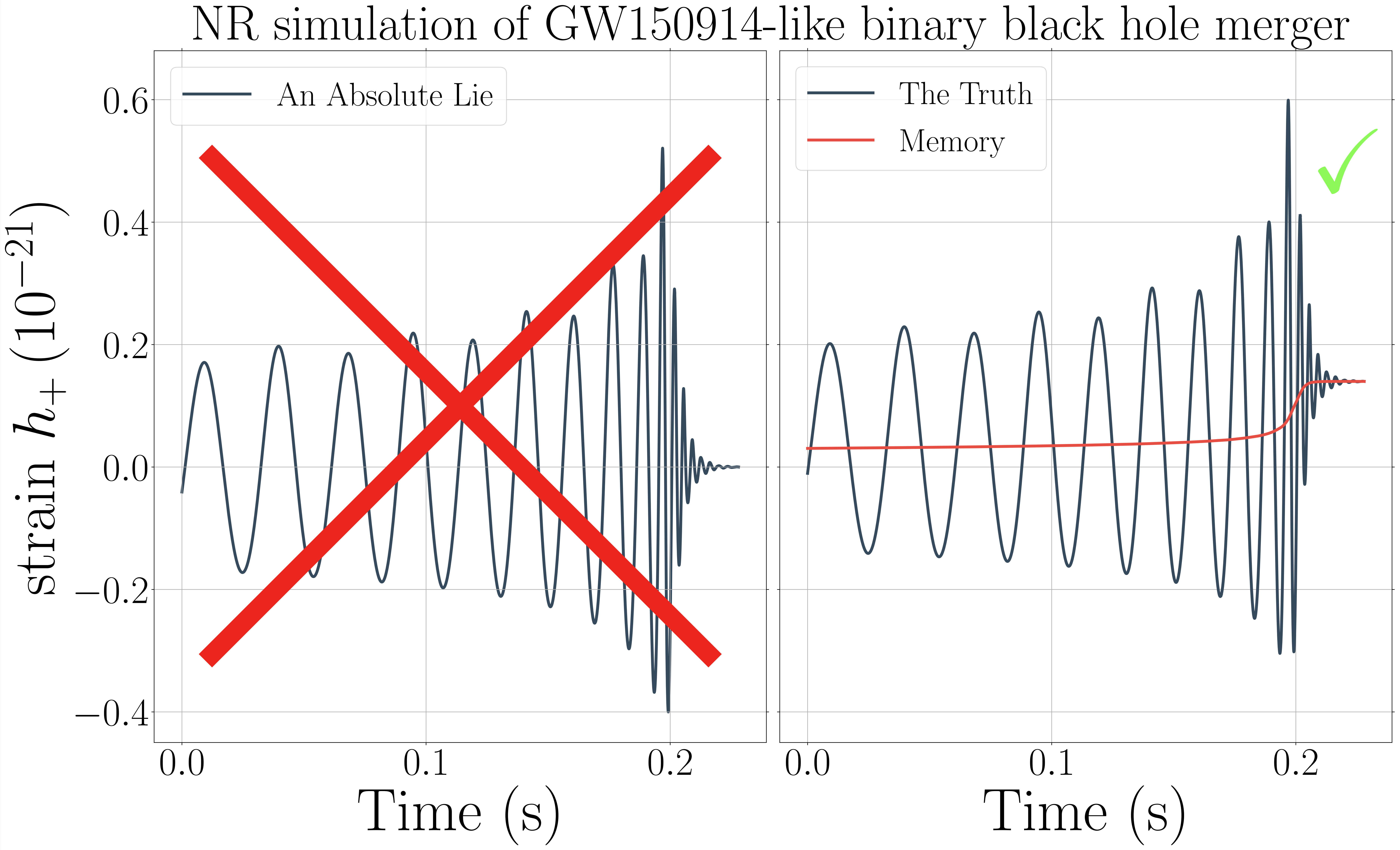
What the result of a NR simulation looks like



What the *memory* in a NR simulation looks like



~~Wow! Our First Detection!~~ The Truth!



The BMS Group

The Hard Way:

- Find an equivalence class of vector fields $\overrightarrow{\xi}$ satisfying Killing's equation $\mathscr{L}_{\overrightarrow{\xi}} g_{ab} = 0$ (approximately, i.e., w.r.t. fall-off conditions) as one approaches future null infinity $\mathscr{I}^+$

The BMS Group

The Hard Way:

- Find an equivalence class of vector fields $\overrightarrow{\xi}$ satisfying Killing's equation $\mathscr{L}_{\overrightarrow{\xi}}g_{ab}=0$ (approximately, i.e., w.r.t. fall-off conditions) as one approaches future null infinity $\mathscr{I}^{+}$

- Yields

$$\overrightarrow{\xi}=\left[\alpha\left(\theta^{A}\right)+\frac{1}{2}uD_{A}Y^{A}\left(\theta^{B}\right)\right]\partial_{u}+Y^{A}\left(\theta_{B}\right)\partial_{A}$$

where

$$Y^{A}=D^{A}\chi+\epsilon^{AB}D_{B}\kappa=\text{boost}+\text{rotation}$$

The BMS Group

The Hard Way:

- Find an equivalence class of vector fields $\vec{\xi}$ satisfying Killing's equation $\mathscr{L}_{\vec{\xi}} g_{ab} = 0$ (approximately, i.e., w.r.t. fall-off conditions) as one approaches future null infinity $\mathscr{I}^+$

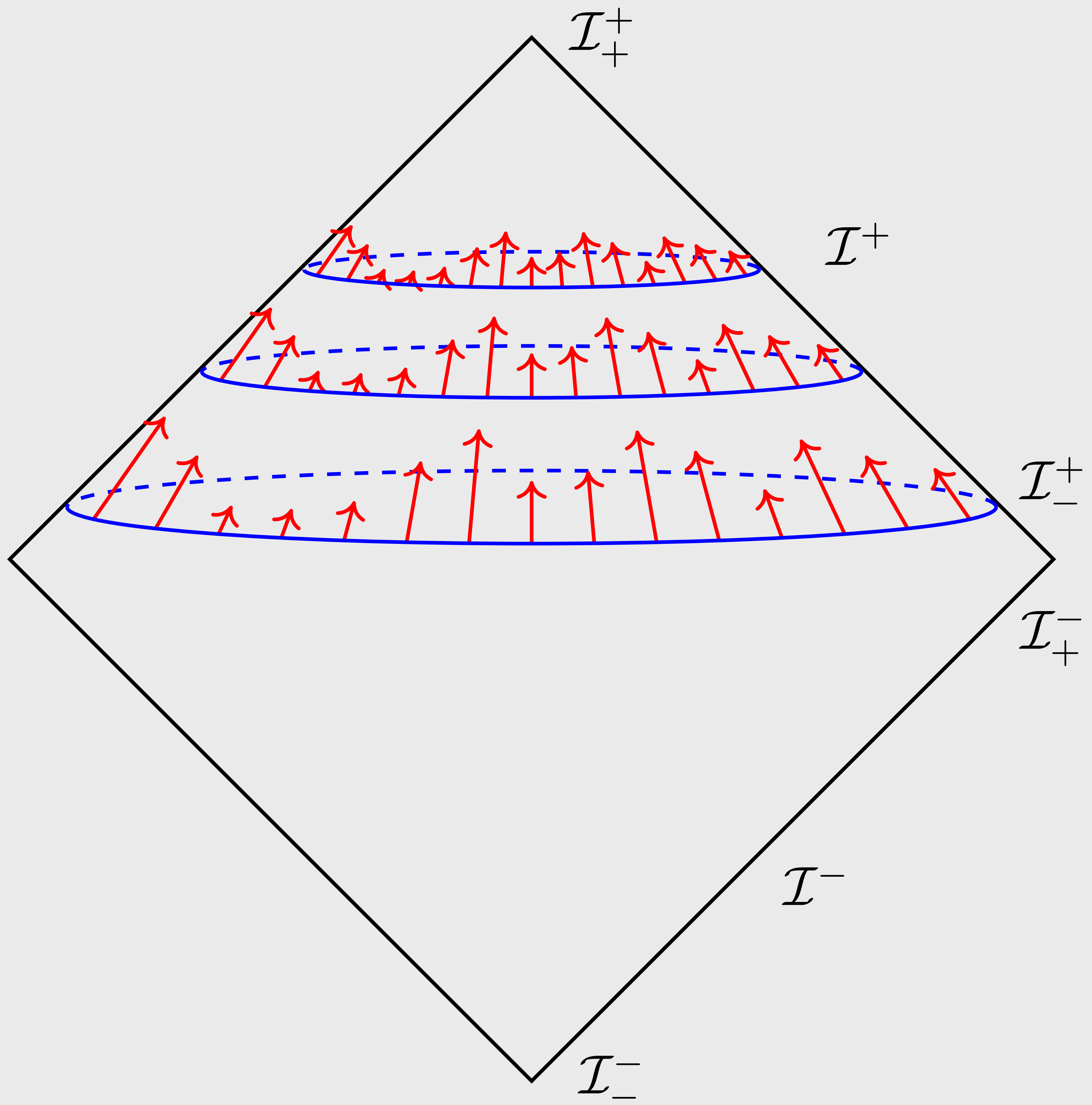
- Yields

$$\vec{\xi} = \left[\alpha\left(\theta^A\right) + \frac{1}{2} u D_A Y^A \left(\theta^B\right)\right] \partial_u + Y^A \left(\theta_B\right) \partial_A$$

where
$Y^A = D^A \chi + \epsilon^{AB} D_B \kappa = \text{boost} + \text{rotation}$

The Easy Way:

- Consider a collection of observers on the celestial two sphere



The BMS Charges and Fluxes

Wald-Zoupas/Dray Charges:

$$Q_{\vec{\xi}} = \frac{1}{4\pi}\int d^2\Omega \left[\alpha m + \frac{1}{2}Y^A \hat{N}_A\right]$$

where

$$m \equiv -\operatorname{Re}\left[\psi_2 + \sigma\dot{\bar{\sigma}}\right]$$

$$\hat{N} \equiv -\left(\psi_1 + \sigma\eth\bar{\sigma} + \frac{1}{2}\eth\left(\sigma\bar{\sigma}\right) + u\eth m\right)$$

This choice of charges corresponds to:

- no fluxes in Minkowski space
- real-valued supermomentum

The BMS Charges and Fluxes

Wald-Zoupas/Dray Charges:

$$Q_{\vec{\xi}} = \frac{1}{4\pi}\int d^2\Omega \left[\alpha m + \frac{1}{2}Y^A \hat{N}_A\right]$$

where

$$m \equiv -\,\mathrm{Re}\left[\psi_2 + \sigma\dot{\bar{\sigma}}\right]$$

$$\hat{N} \equiv -\left(\psi_1 + \sigma\eth\bar{\sigma} + \frac{1}{2}\eth\left(\sigma\bar{\sigma}\right) + u\eth m\right)$$

This choice of charges corresponds to:

- no fluxes in Minkowski space
- real-valued supermomentum

Balance Laws:

For u_0 a non-radiative section of $\mathscr{I}^+$,

$$\mathrm{Re}\left[\bar{\eth}^2\sigma\right] = \mathrm{Re}\left[m + \mathscr{E} + \left(\bar{\eth}^2\sigma - m\right)|^{u_0}\right]$$

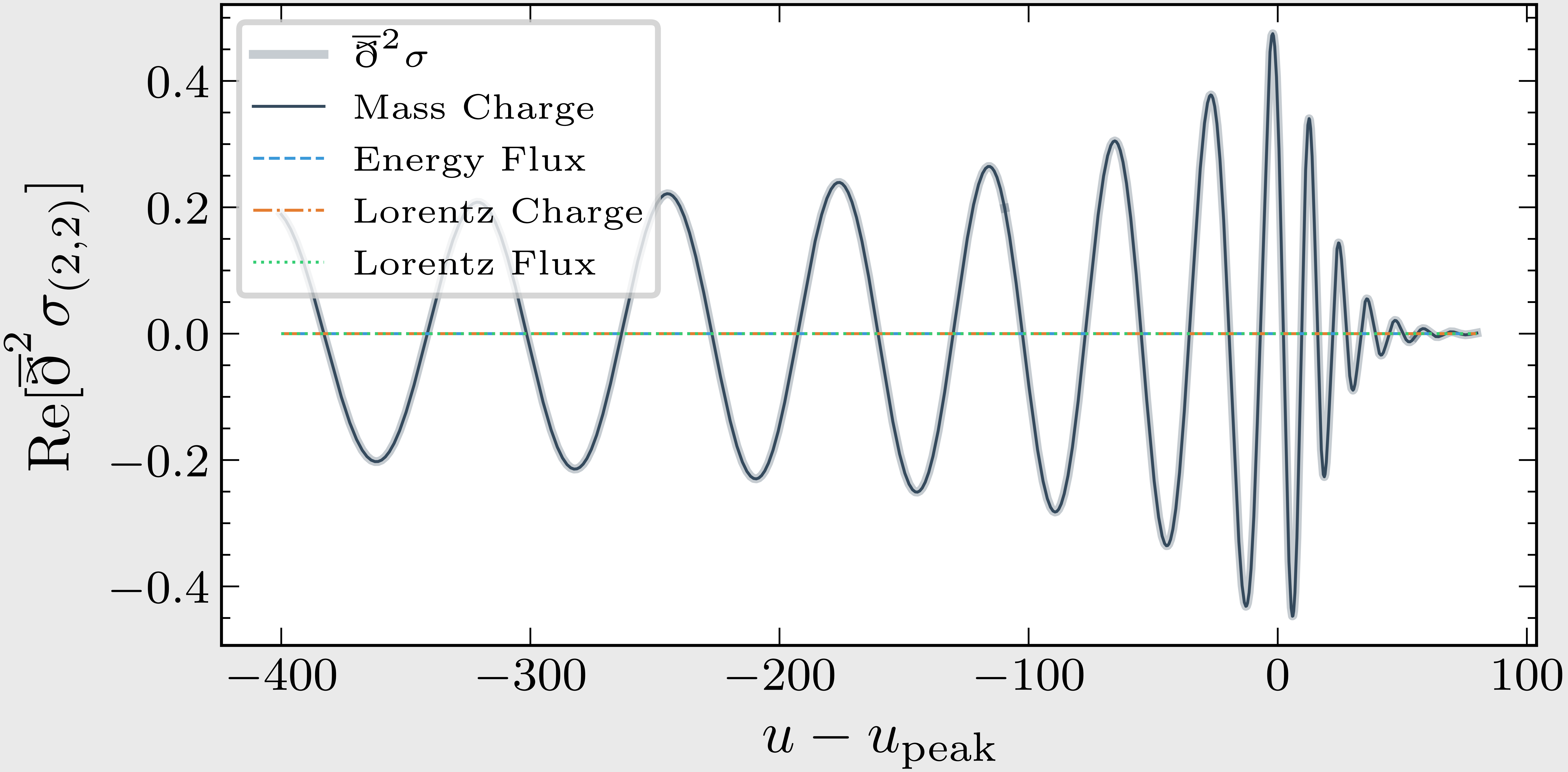
$$\mathrm{Im}\left[\bar{\eth}^2\sigma\right] = -\frac{d}{du}\left(\eth\bar{\eth}\right)^{-1}\mathrm{Im}\left[\bar{\eth}\left(\hat{N} + \mathscr{J}\right)\right]$$

where

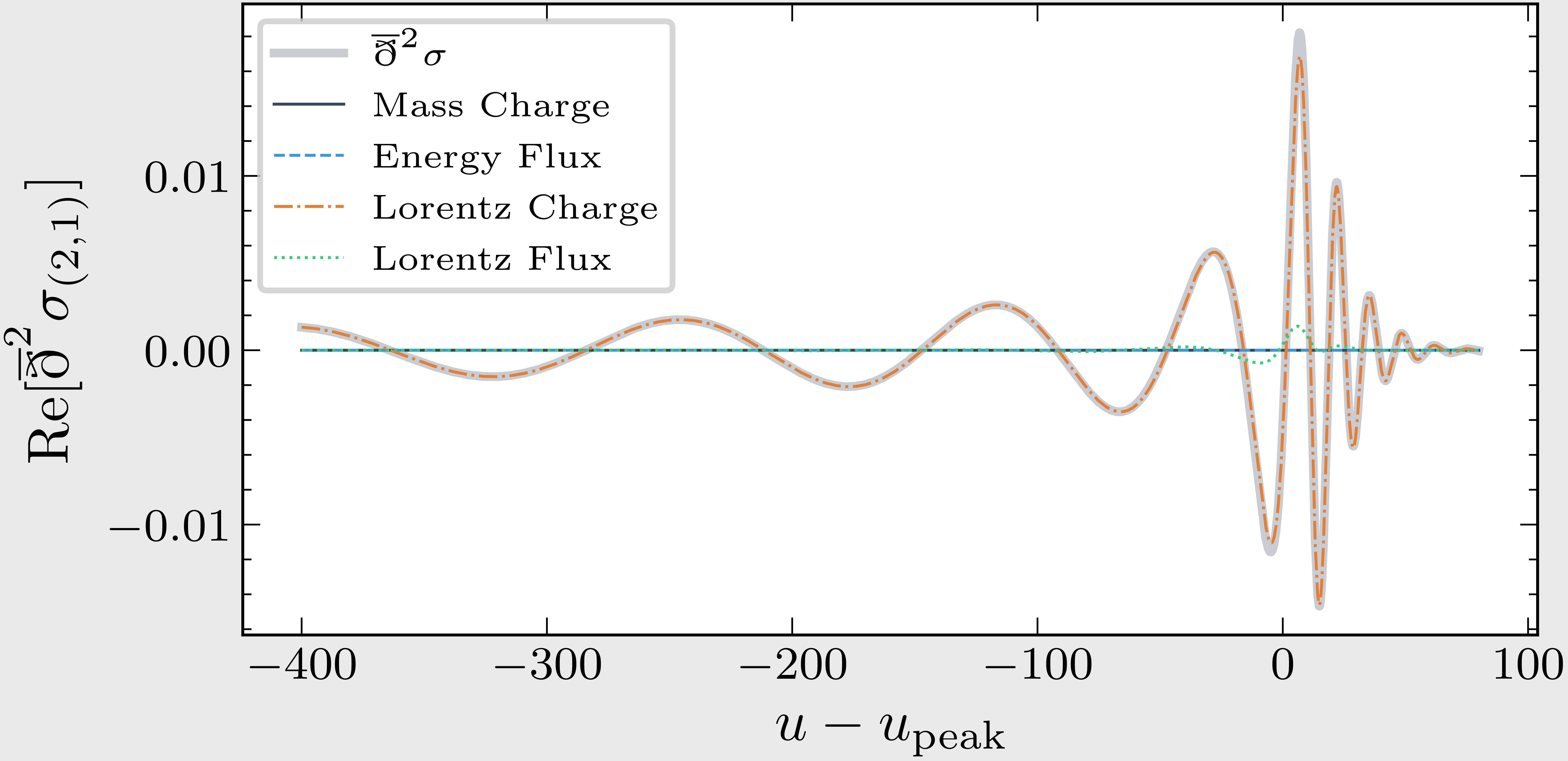
$$\mathscr{E} \equiv \int_{u_0}^{u} |\dot{\sigma}|^2\,du$$

$$\mathscr{J} \equiv \frac{1}{2}\int_{u_0}^{u}\left(3\dot{\sigma}\eth\bar{\sigma} - 3\sigma\eth\dot{\bar{\sigma}} + \bar{\sigma}\eth\dot{\sigma} - \dot{\bar{\sigma}}\eth\sigma\right)du$$

Numerical Calculation of BMS Charges (Bondi Mass Aspect)



Numerical Calculation of BMS Charges (Lorentz Aspect)



The BMS Charges and Fluxes

Wald-Zoupas/Dray Charges:

$$Q_{\vec{\xi}} = \frac{1}{4\pi}\int d^2\Omega \left[\alpha m + \frac{1}{2}Y^A \hat{N}_A\right]$$

where

$$m \equiv -\operatorname{Re}\left[\psi_2 + \sigma\dot{\bar{\sigma}}\right]$$

$$\hat{N} \equiv -\left(\psi_1 + \sigma\eth\bar{\sigma} + \frac{1}{2}\eth\left(\sigma\bar{\sigma}\right) + u\eth m\right)$$

This choice of charges corresponds to:

- no fluxes in Minkowski space
- real-valued supermomentum

Balance Laws:

For u_0 a non-radiative section of $\mathscr{I}^+$,

$$\operatorname{Re}\left[\bar{\eth}^2\sigma\right] = \operatorname{Re}\left[m + \mathscr{E} + \left(\bar{\eth}^2\sigma - m\right)|^{u_0}\right]$$

$$\operatorname{Im}\left[\bar{\eth}^2\sigma\right] = -\frac{d}{du}\left(\eth\bar{\eth}\right)^{-1}\operatorname{Im}\left[\bar{\eth}\left(\hat{N} + \mathscr{J}\right)\right]$$

where

$$\mathscr{E} \equiv \int_{u_0}^{u} |\dot{\sigma}|^2 du$$

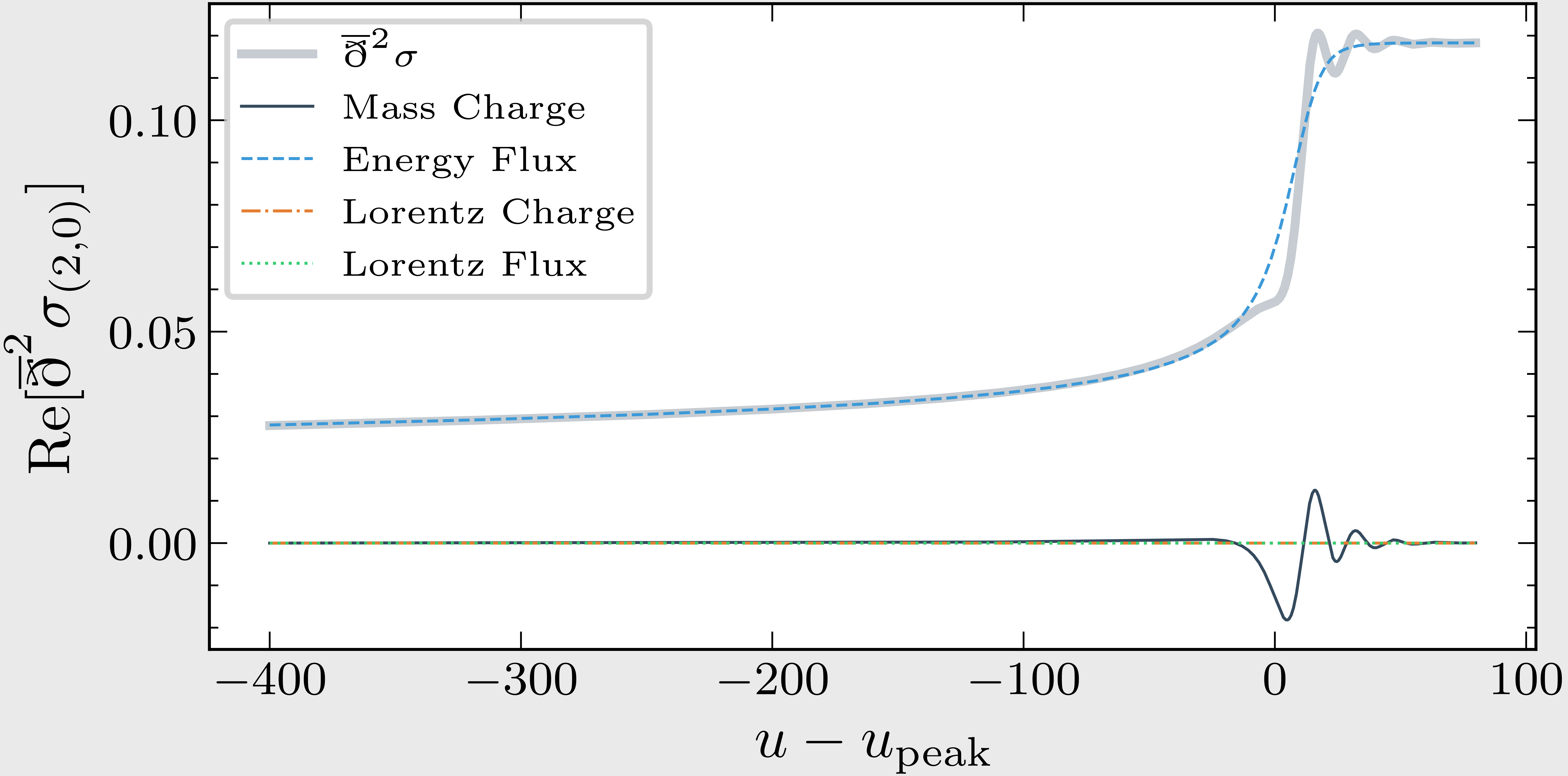
$$\mathscr{J} \equiv \frac{1}{2}\int_{u_0}^{u}\left(3\dot{\sigma}\eth\bar{\sigma} - 3\sigma\eth\dot{\bar{\sigma}} + \bar{\sigma}\eth\dot{\sigma} - \dot{\bar{\sigma}}\eth\sigma\right)du$$

The Gravitational Memory Effect(s)

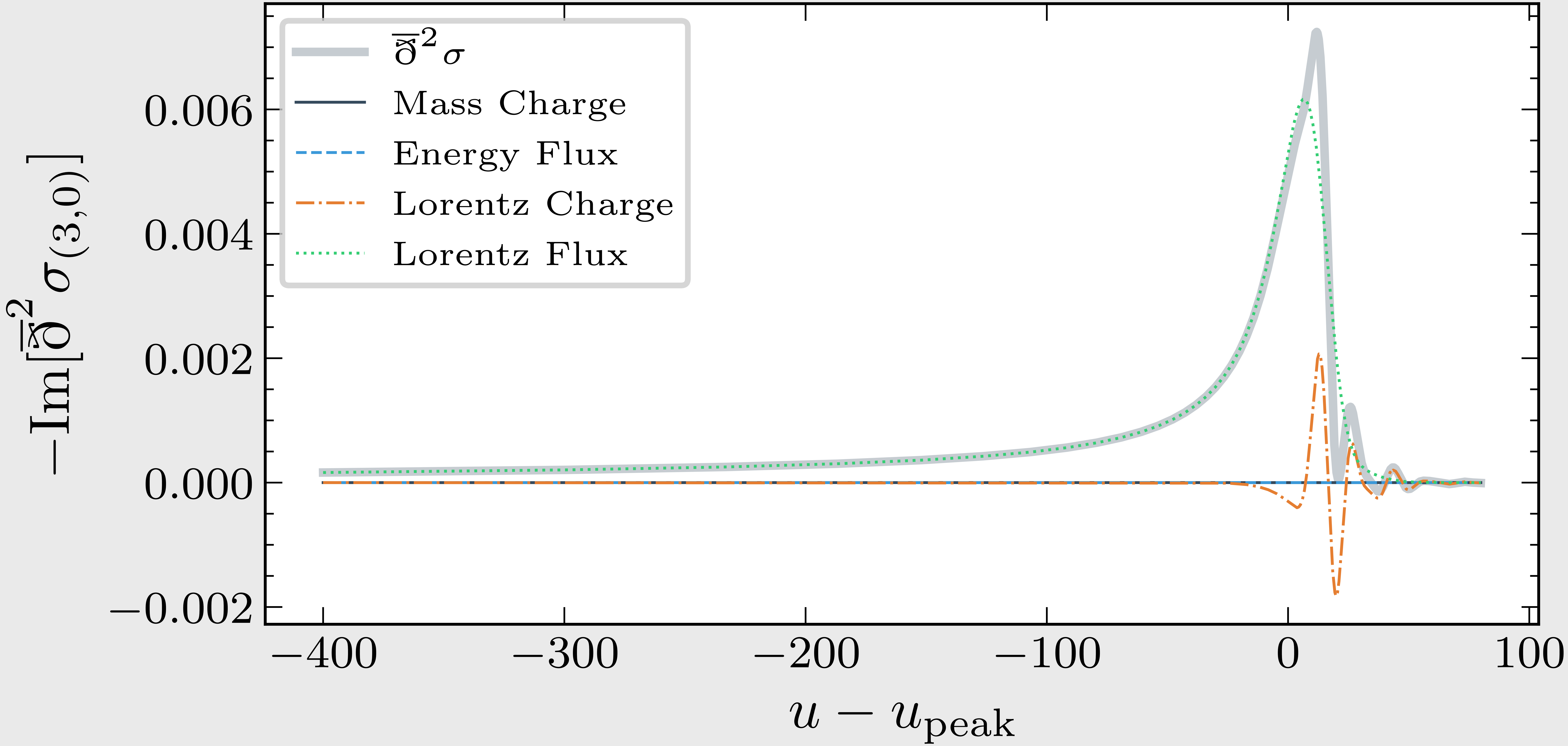
$$\Delta\mathrm{Re}\left[\sigma\right]=\eth^2\left(\eth^2\bar{\eth}^2\right)^{-1}\mathrm{Re}\left[\Delta m+\mathscr{E}\right],\quad\int_{-\infty}^{+\infty}\mathrm{Im}\left[\sigma\right]du=-\,\eth^2\left(\eth^2\bar{\eth}^2\right)^{-1}\left(\eth\bar{\eth}\right)^{-1}\mathrm{Im}\left[\bar{\eth}\left(\Delta\hat{N}+\mathscr{J}\right)\right]$$

Observable	Parity	Type	Memory
Bondi Mass Aspect	Electric	Ordinary (linear)	Displacement
Energy Flux	Electric	Null (non-linear)	Displacement
Lorentz Aspect	Electric/Magnetic	Ordinary (linear)	Center-of-Mass/Spin
Lorentz Flux	Electric/Magnetic	Null (non-linear)	Center-of-Mass/Spin

Numerical Calculation of Memory Effects (Energy Flux)



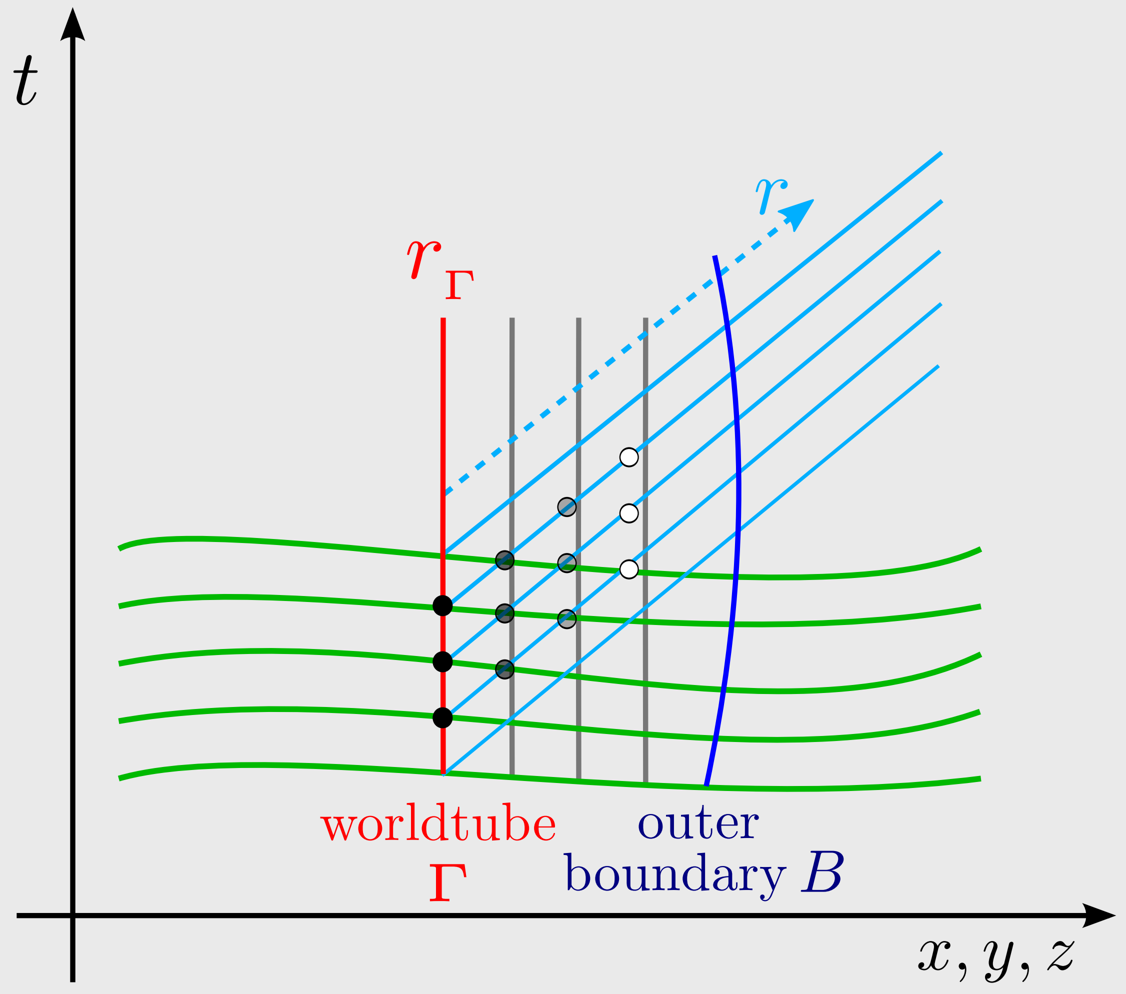
Numerical Calculation of Memory Effects (Lorentz Flux)



Extrapolation vs. Cauchy-characteristic Extraction

Extrapolation:

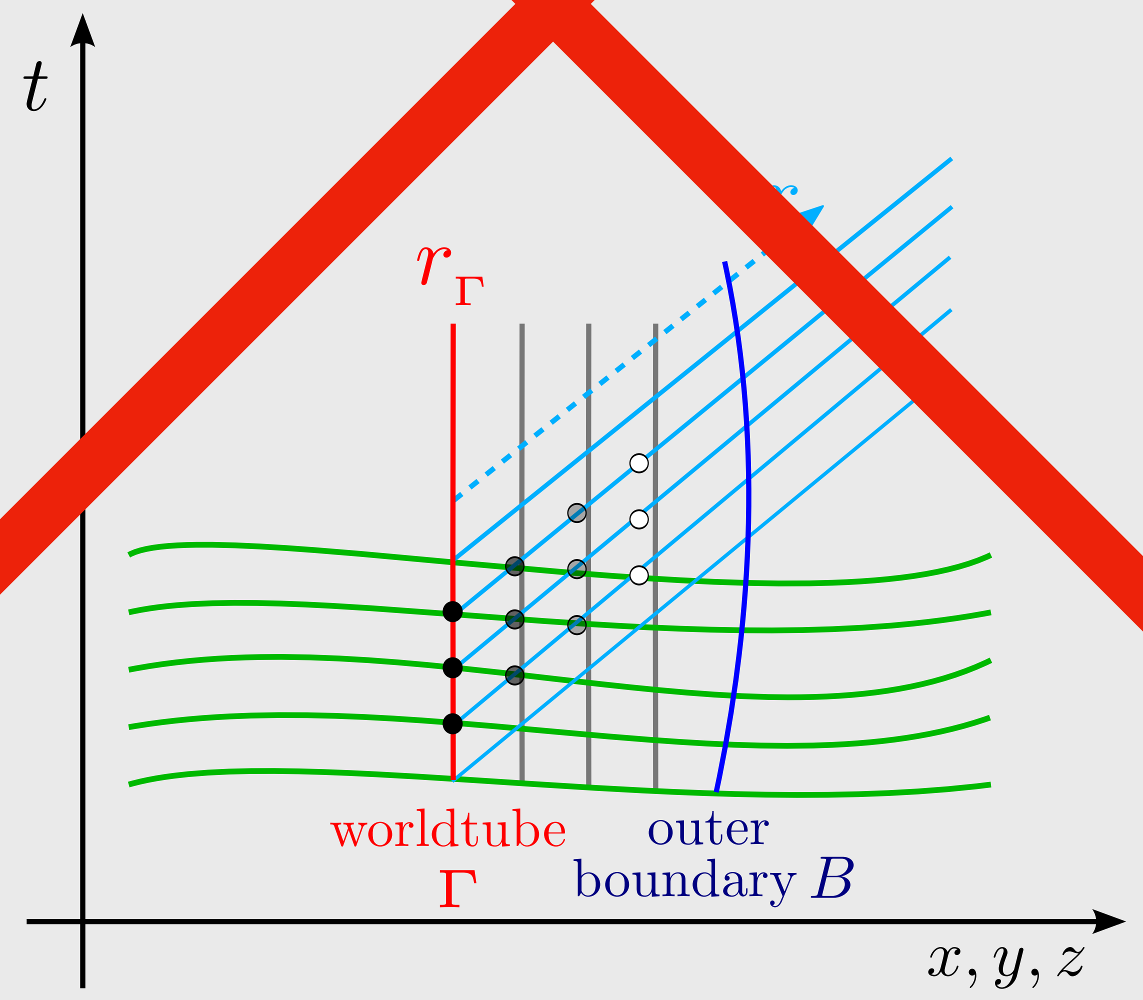
- Obtain the metric (and derivatives) on a series of finite-radius world-tubes
- Interpolate (radially) between various points on the radially-varying world tubes
- Extrapolate to $r \to \infty$



Extrapolation vs. Cauchy-characteristic Extraction

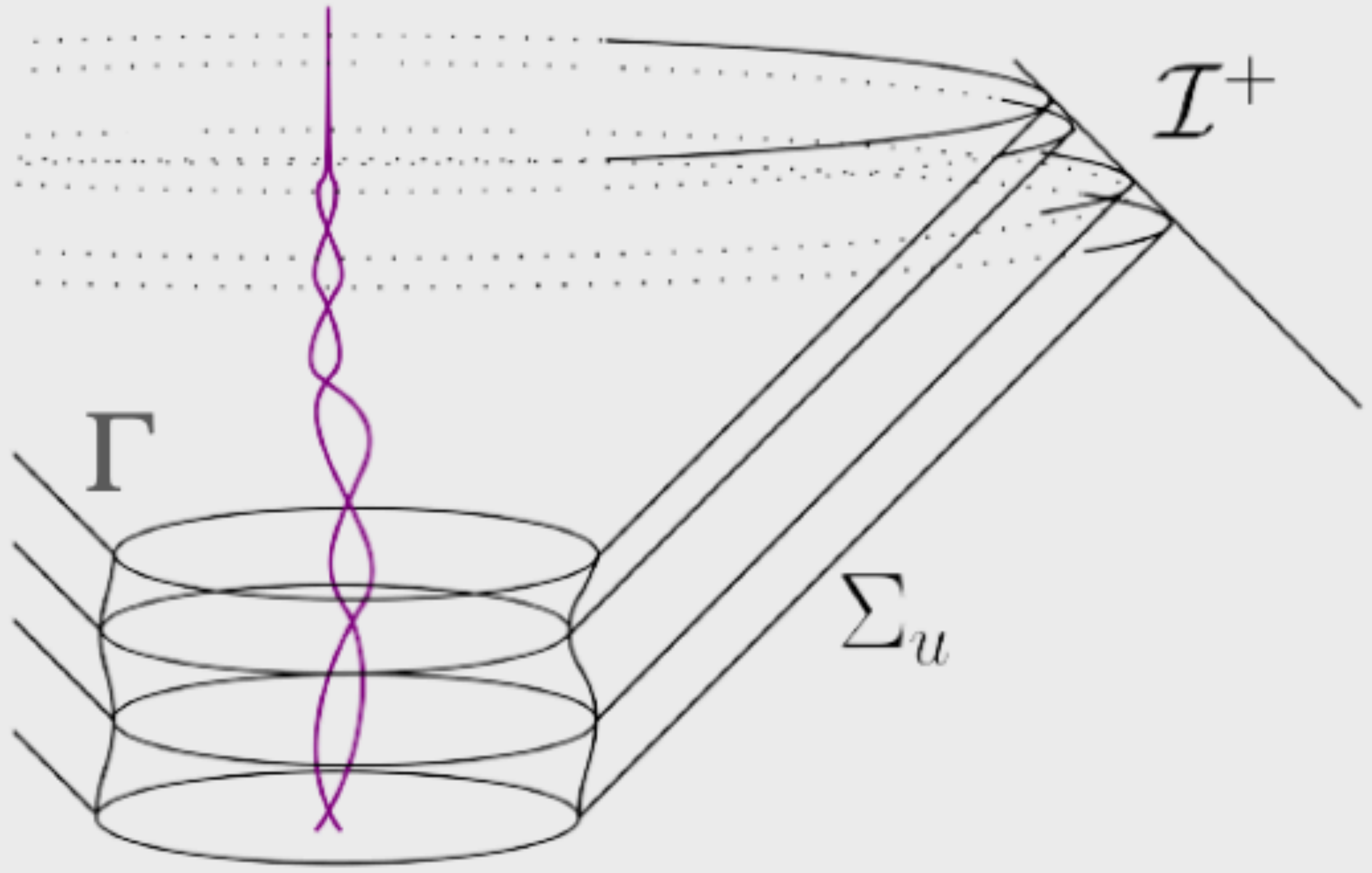
Extrapolation:

- Obtain the metric (and derivatives) on a series of finite-radius world-tubes
- Interpolate (radially) between various points on the radially varying world-tubes
- Extrapolate to $r \to \infty$



Cauchy-characteristic Extraction ✓

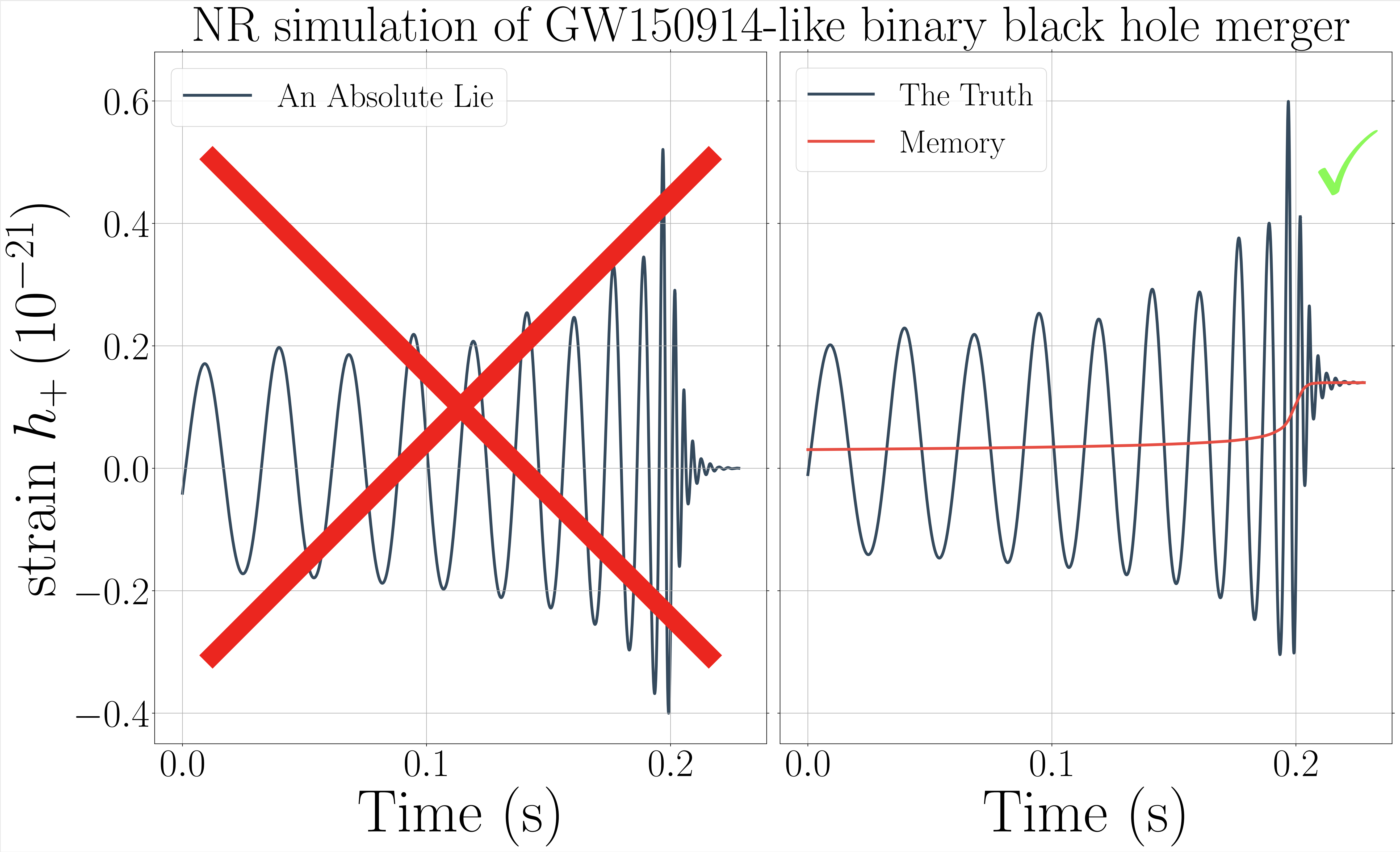
- Obtain the metric (and derivatives) on a finite-radius world-tube Γ
- Initialize the first null hypersurface Σ_u by matching the strain (and derivatives) at Γ
- Evolve Σ_u forward in time



Looking toward the Future

Future prospects with memory:

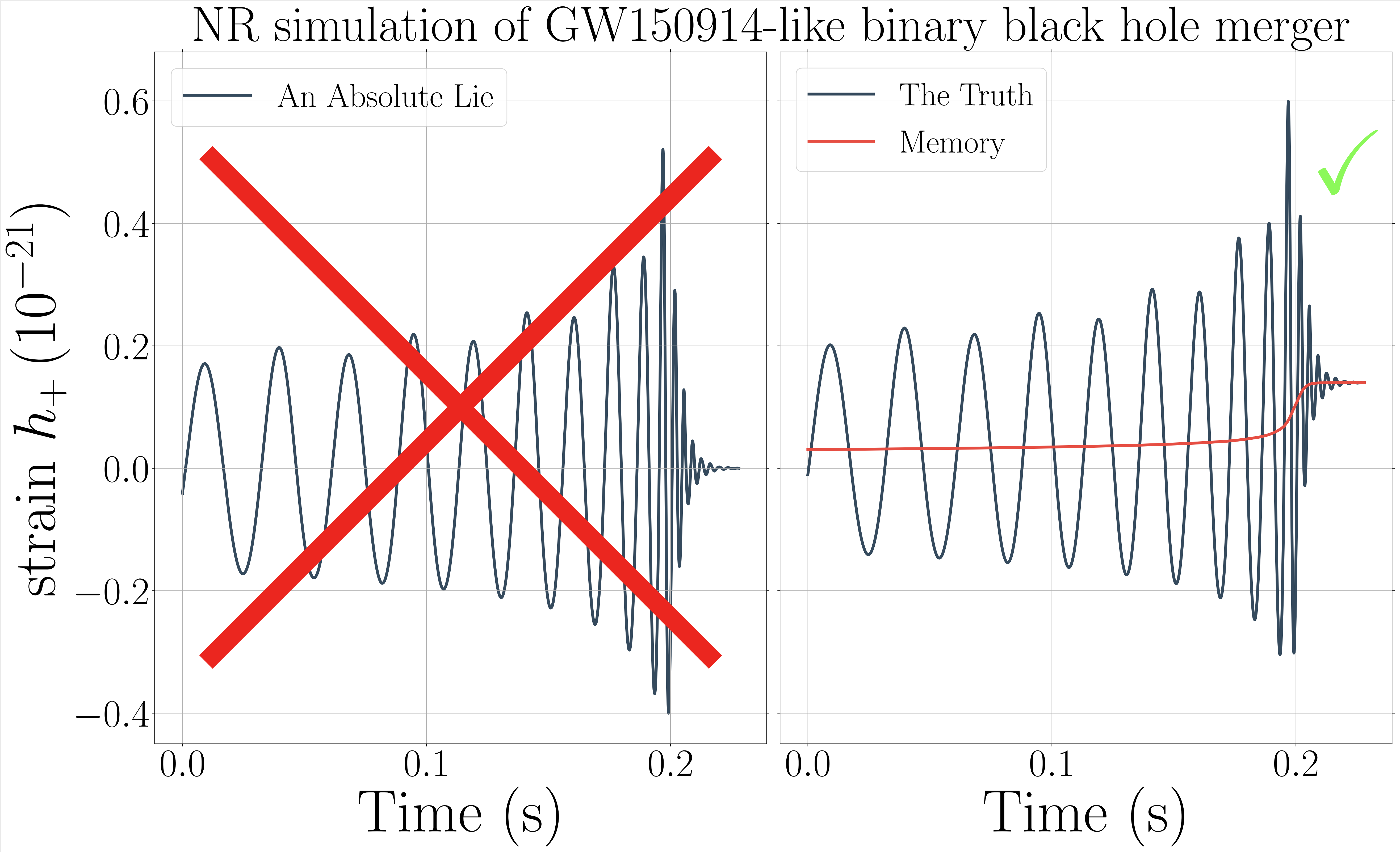
- Build more correct NR waveforms



Looking toward the Future

Future prospects with memory:

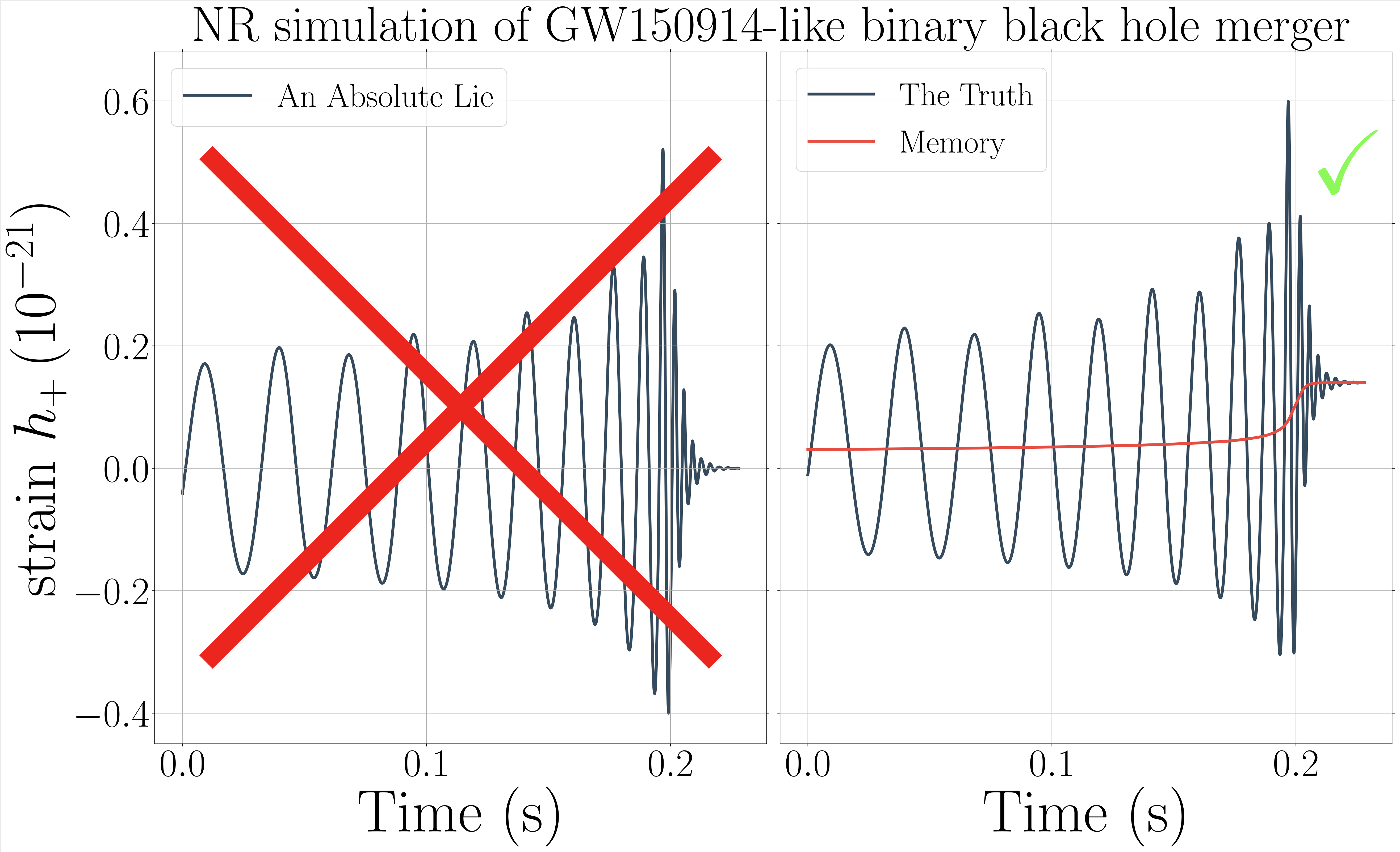
- Build more correct NR waveforms
- Detect memory effects



Looking toward the Future

Future prospects with memory:

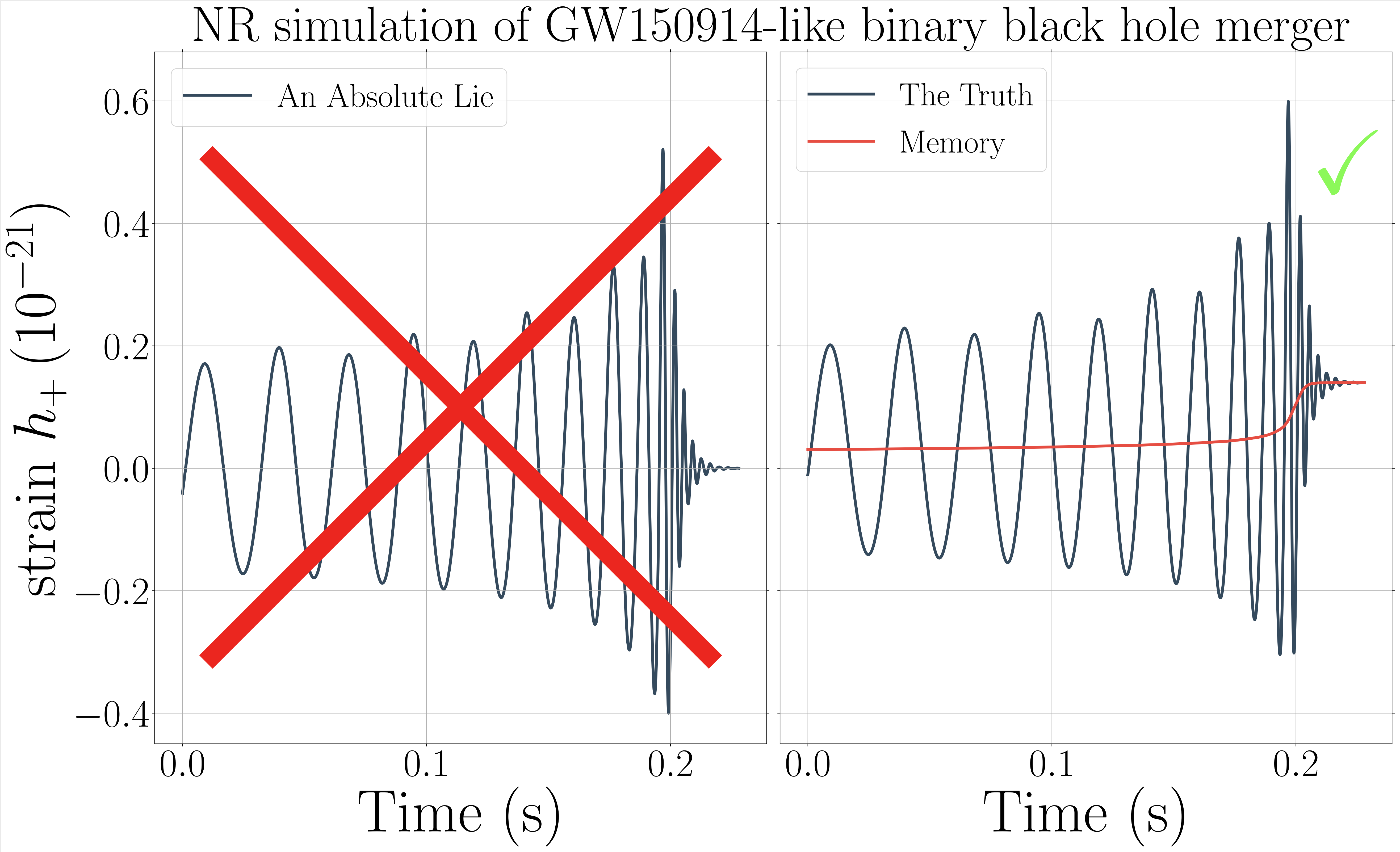
- Build more correct NR waveforms
- Detect memory effects
- Detect supertranslated BHs



Looking toward the Future

Future prospects with memory:

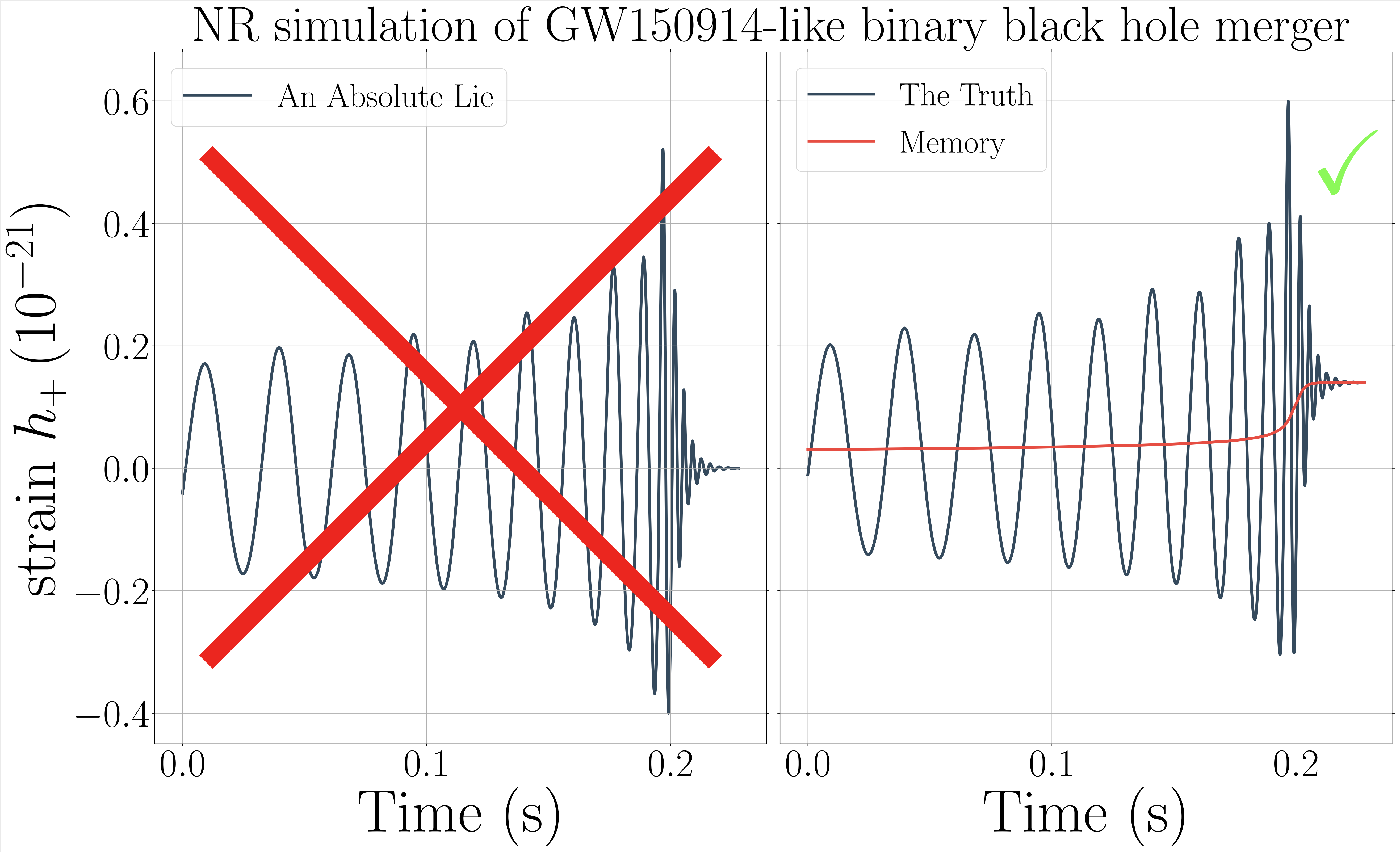
- Build more correct NR waveforms
- Detect memory effects
- Detect supertranslated BHs
- Detect BH memory



Looking toward the Future

Future prospects with memory:

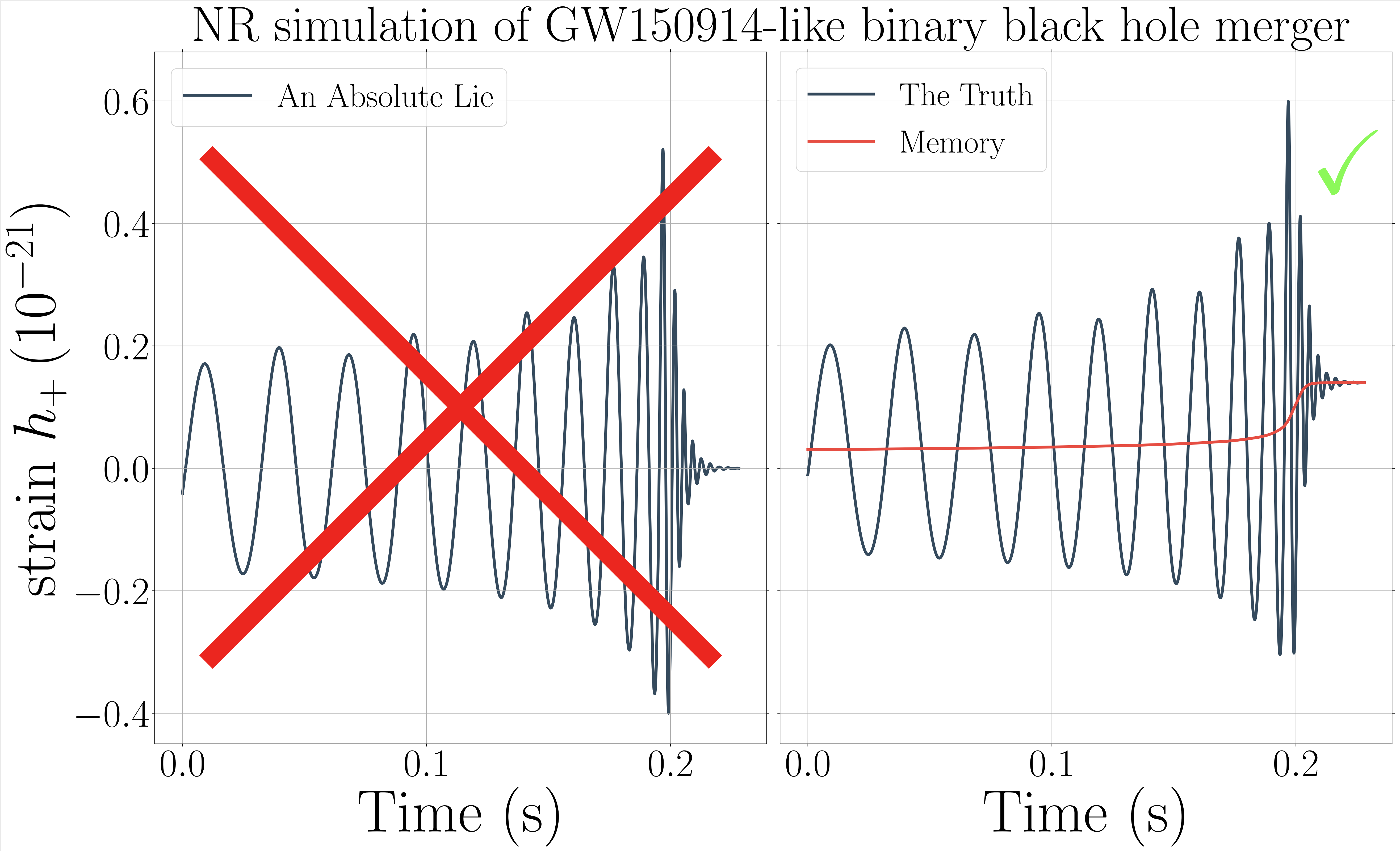
- Build more correct NR waveforms
- Detect memory effects
- Detect supertranslated BHs
- Detect BH memory
- Understand asymptotic symmetries



Looking toward the Future

Future prospects with memory:

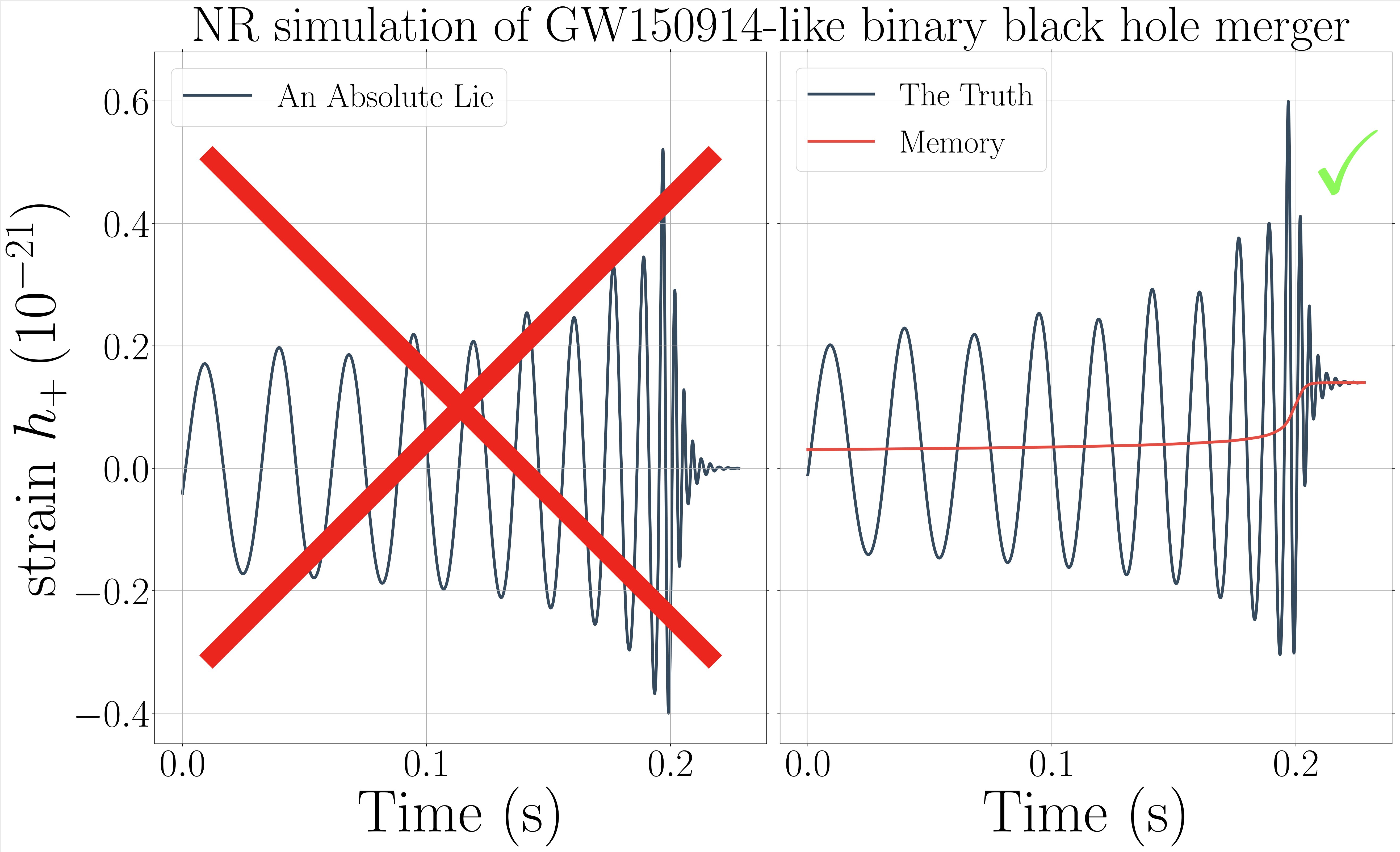
- Build more correct NR waveforms
- Detect memory effects
- Detect supertranslated BHs
- Detect BH memory
- Understand asymptotic symmetries
- An answer to the BH information paradox



Looking toward the Future

Future prospects with memory:

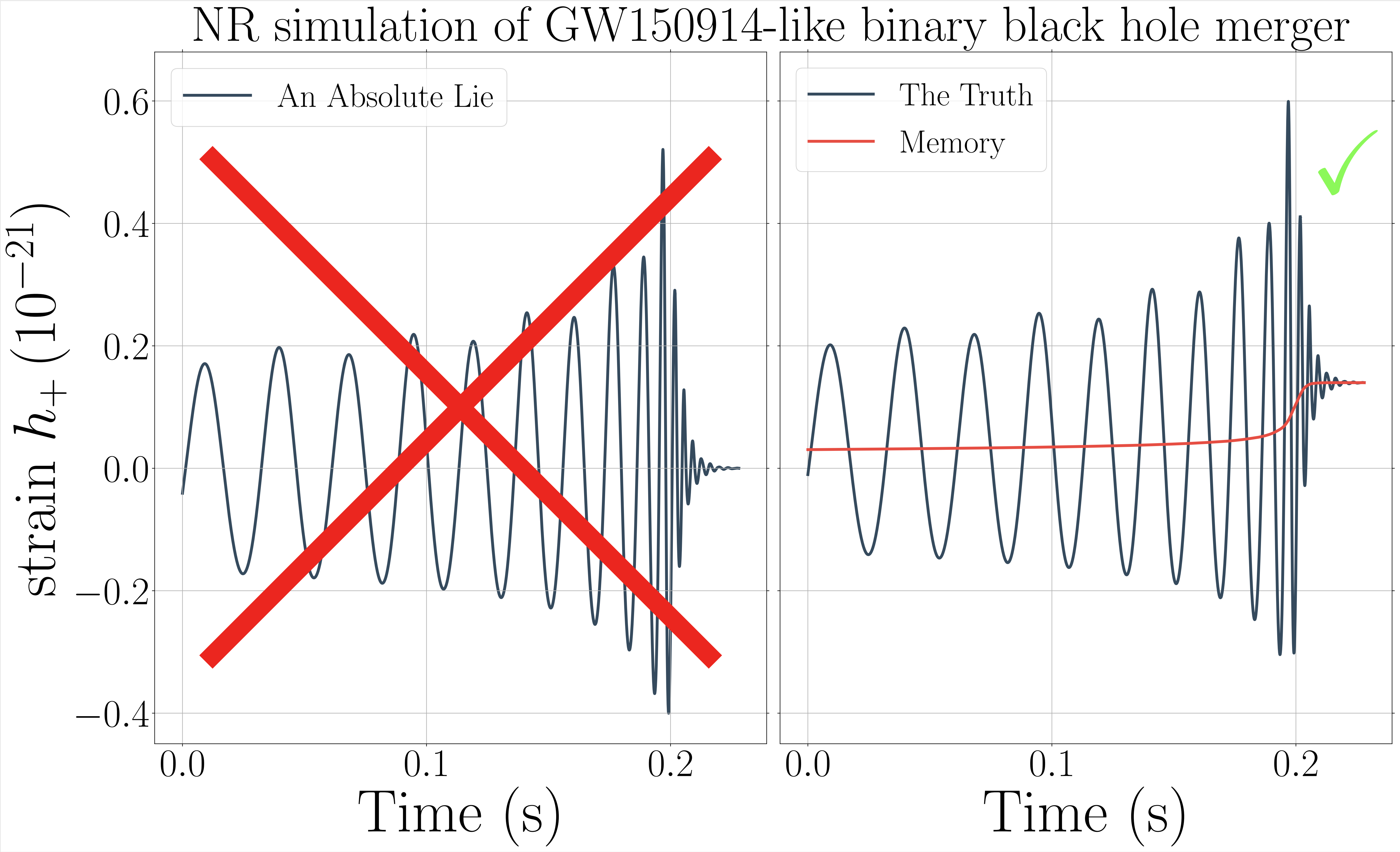
- Build more correct NR waveforms
- Detect memory effects
- Detect supertranslated BHs
- Detect BH memory
- Understand asymptotic symmetries
- An answer to the BH information paradox
- Make Quantum Gravity (QG) harder



Looking toward the Future

Future prospects with memory:

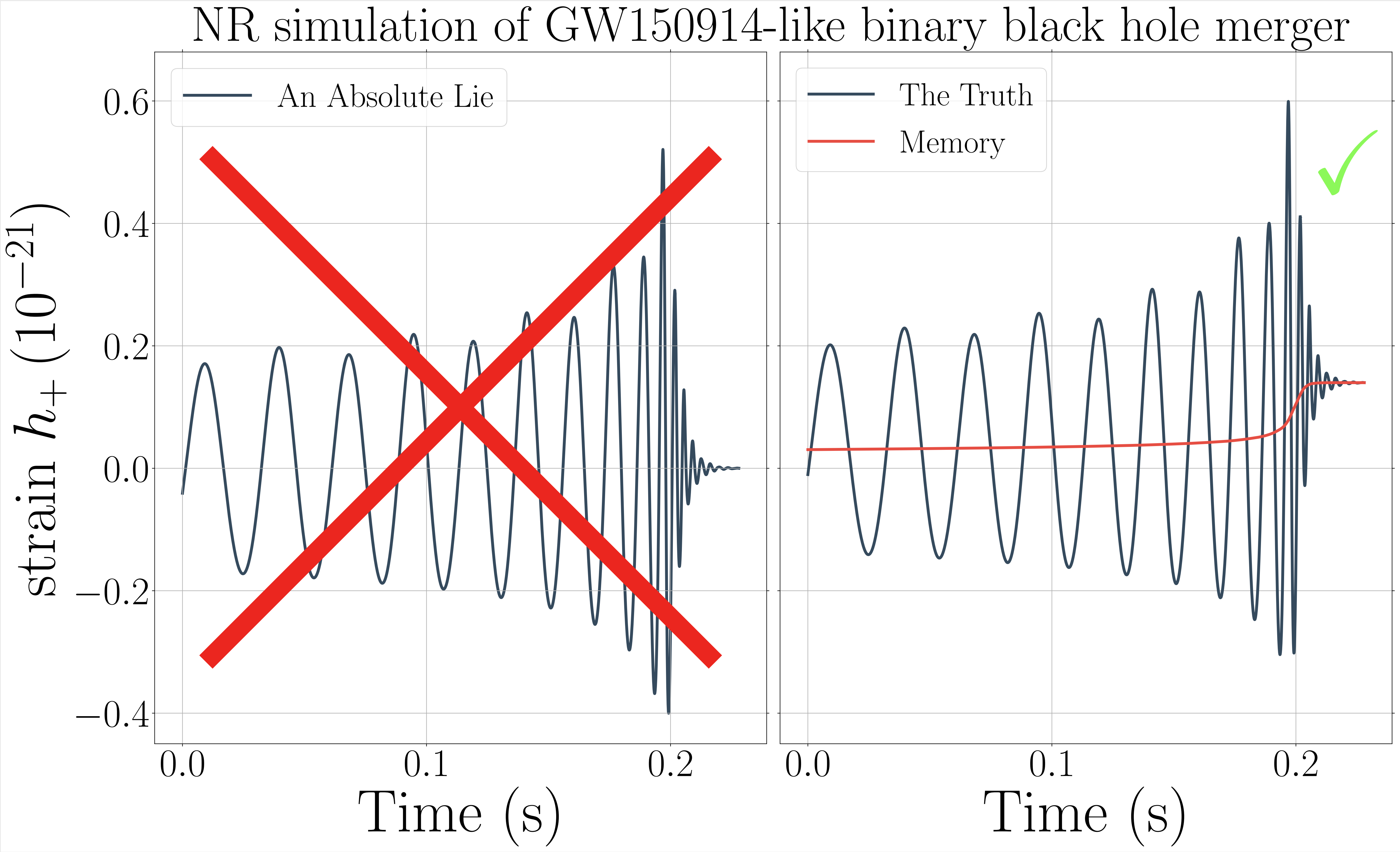
- Build more correct NR waveforms
- Detect memory effects
- Detect supertranslated BHs
- Detect BH memory
- Understand asymptotic symmetries
- An answer to the BH information paradox
- Make Quantum Gravity (QG) harder
- Make Quantum Gravity easier



Looking toward the Future

Future prospects with memory:

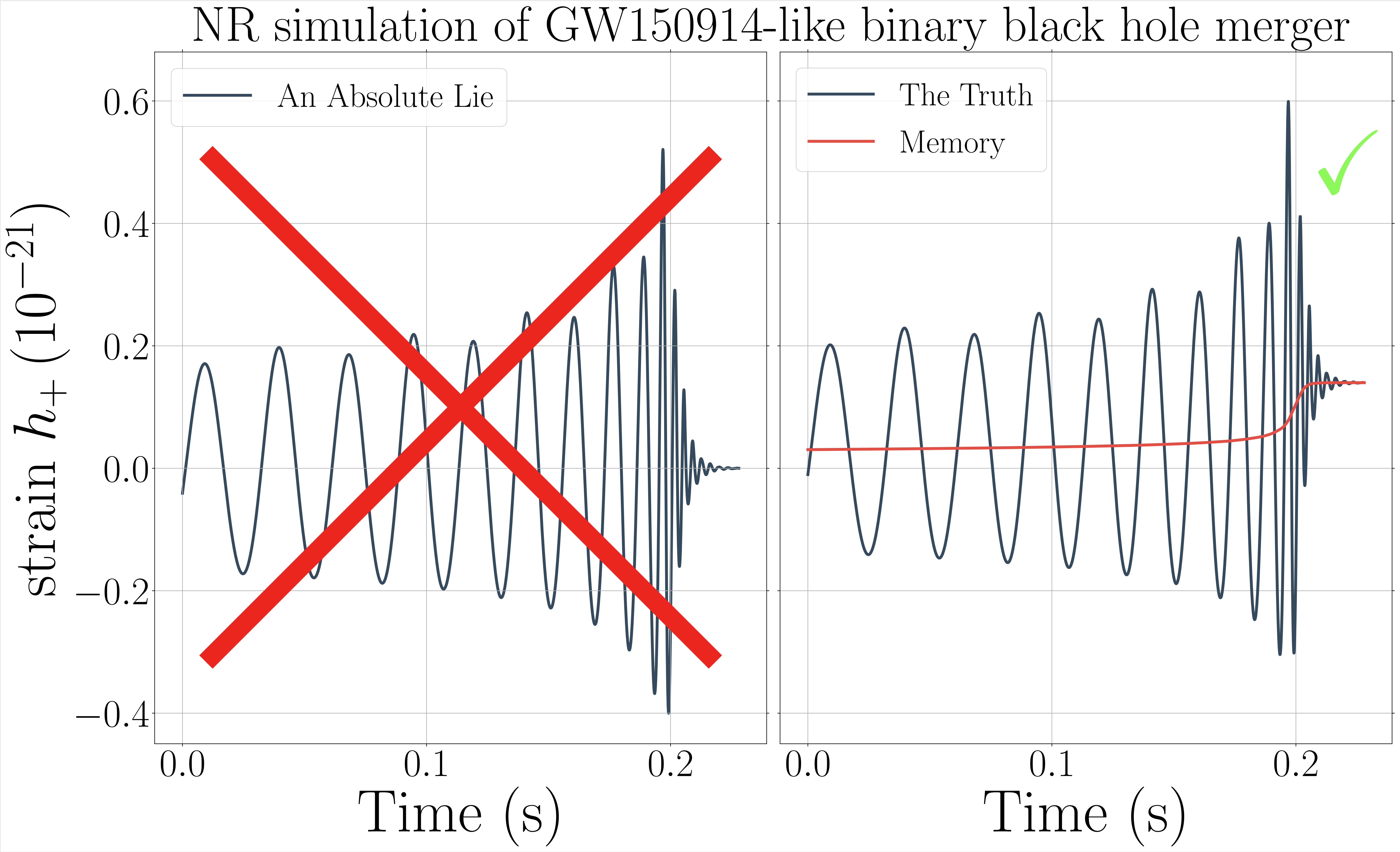
- Build more correct NR waveforms
- Detect memory effects
- Detect supertranslated BHs
- Detect BH memory
- Understand asymptotic symmetries
- An answer to the BH information paradox
- Make Quantum Gravity (QG) harder
- Make Quantum Gravity easier
- Understand soft theorems for QG



Looking toward the Future

Future prospects with memory:

- Build more correct NR waveforms
- Detect memory effects
- Detect supertranslated BHs
- Detect BH memory
- Understand asymptotic symmetries
- An answer to the BH information paradox
- Make Quantum Gravity (QG) harder
- Make Quantum Gravity easier
- Understand soft theorems for QG
- etc.



Conclusion

Conclusion

- May this talk forever permeate your (Gravitational) Memory

Thank you!