Cosmic Vibes: The Stochastic GW Background

ARC Seminar - 21 February 2025

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Outline

- What is the GWB?
- What does the GWB look like?
- How do we currently detect GWB?
- Future prospects for GWB

Intended to be a little more of a broad overview into all the topics!

Why should you care?

Studying the GWB requires:

- detector analysis
- astrophysical intuition
- Bayesian statistics
- General Relativity

Pexels Stock Photo.

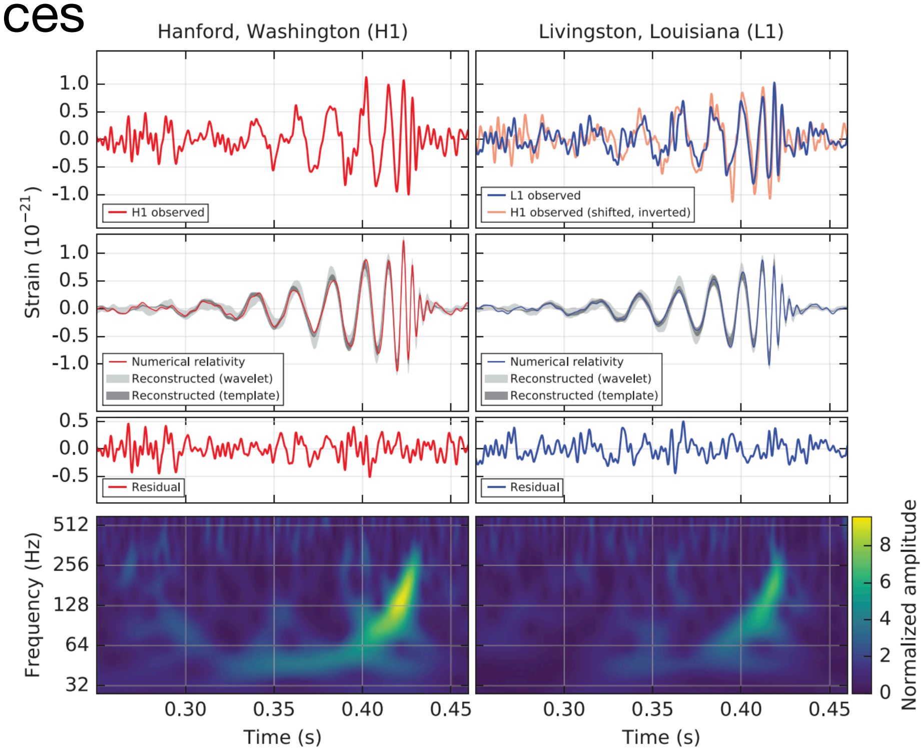
Something for all the experimentalists, instrumentalists, astronomers, theorists, and analysts in the room!

What is a GW?

- perturbations to the spacetime metric from energetic events
- mostly hear of these as compact coalescences
- solution is a sum of plane waves:

$$h_{(\ell,m)}(t) = A^{(\ell,m)}(t)e^{-i\Phi^{(\ell,m)}(t)}$$

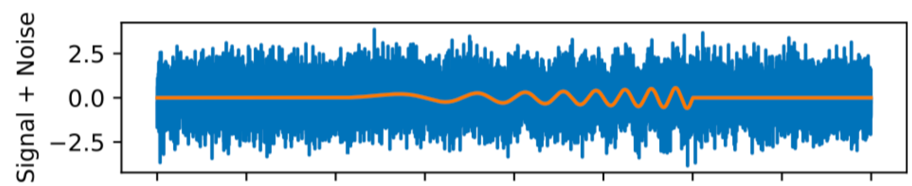
  - (ℓ, m) - harmonic modes
  - $A^{(\ell,m)}$ - real amplitudes
  - $\Phi^{(\ell,m)}$ - real phases



PRL 116, 061102 (2016)

GW Background

- composite signal of unresolved sources
- **treated as stochastic, isotropic, stationary, Gaussian**
    - generally assumed to be unpolarized
- 2 distinct generation mechanisms:
    - superposition of astrophysical signals
    - cosmological and primordial sources
        - inflationary era vacuum fluctuations
        - phase transitions, nonlinear phenomena



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Calculating the GWB

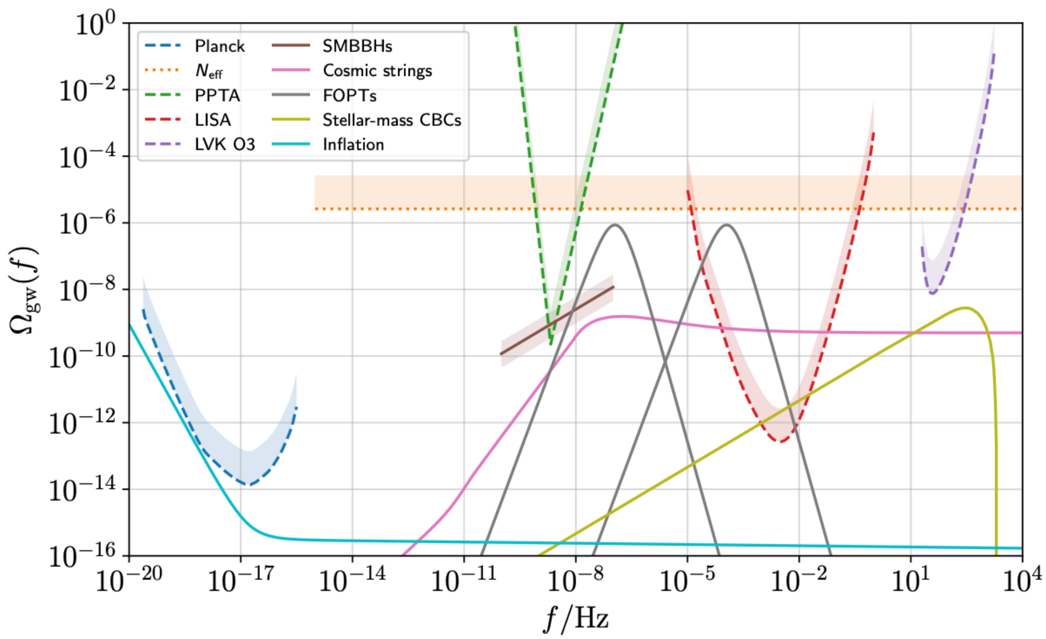
From CMB and BBN data, we determine upper bound:

$$\Omega_{\mathrm{GW}} < 1.2 \times 10^{-6} \left(\frac{H_0}{100\,\mathrm{km/s/Mpc}} \right)^{-2}$$

This is the **total** GW energy density.

Energy density frequency spectrum:

$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_c} \frac{d\rho_{\mathrm{GW}}(f)}{d\ln f}$$



Renzini, et. al. (2022)

Calculating the GWB

Homogeneity and anisotropy imply:

$$\rho_{\mathrm{GW}} = \int_0^\infty \frac{df}{f} \int_0^\infty dz \frac{N(z)}{1+z} f_r \frac{dE_{\mathrm{GW}}}{df_r}$$

event rate v. redshift

energy contribution v. frequency bin

Rewriting energy frequency spectrum:

$$\rho_{\mathrm{GW}} = \int_0^\infty \frac{df}{f} \rho_c \Omega_{\mathrm{GW}}(f)$$

Comparing the two above yields:

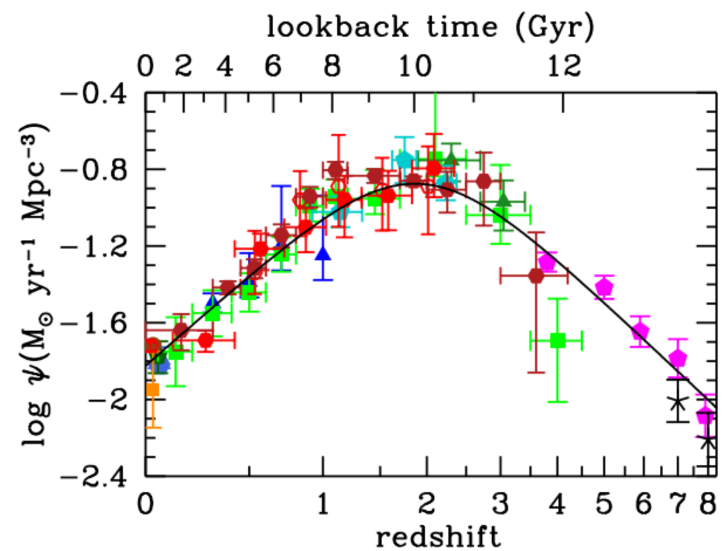
$$\rho_c \Omega_{\mathrm{GW}}(f) = \int_0^\infty dz \frac{N(z)}{1+z} \left[f_r \frac{dE_{\mathrm{GW}}}{df_r} \right]_{f_r = f(1+z)}$$

GWB Sources: Astrophysical Background

For the low-redshift universe, we can rewrite the event rate using the Madau-Dickinson star-formation rate (SFR) (1403.0007):

$$R(z) = \mathscr{C}(\alpha, \beta, z_p) \frac{R_0(1+z)^\alpha}{1 + \left(\frac{1+z}{1+z_p}\right)^\beta}$$

- R_0 - local SFR
- $\mathscr{C}(\alpha, \beta, z_p)$ - normalization factor s.t. $R(z=0) = R_0$
- z_p - redshift where merger rate peaks
- α, β - spectral indices before and after z_p



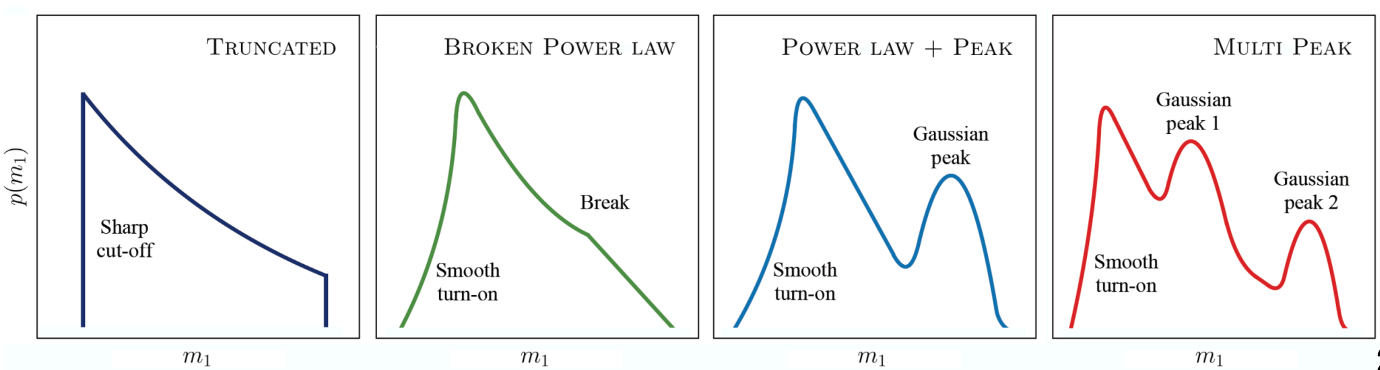
GWB Sources: Astrophysical Background

Averaging the energy per frequency bin over the source populations:

$$\left\langle \frac{dE_{\rm GW}}{df_r} \right\rangle = \int d\theta p(\theta) \frac{dE_{\rm GW}(\theta; f_r)}{df_r}$$

This distribution is informed by the mass distribution:

$$\langle \frac{dE}{df}|_{f(1+z)} \rangle = \int dm_1 dm_2 \frac{dE}{df}(m_1, m_2; f(1+z)) p(m_1, m_2)$$



2010.14533

GWB Sources: Astrophysical Background

Putting everything together:

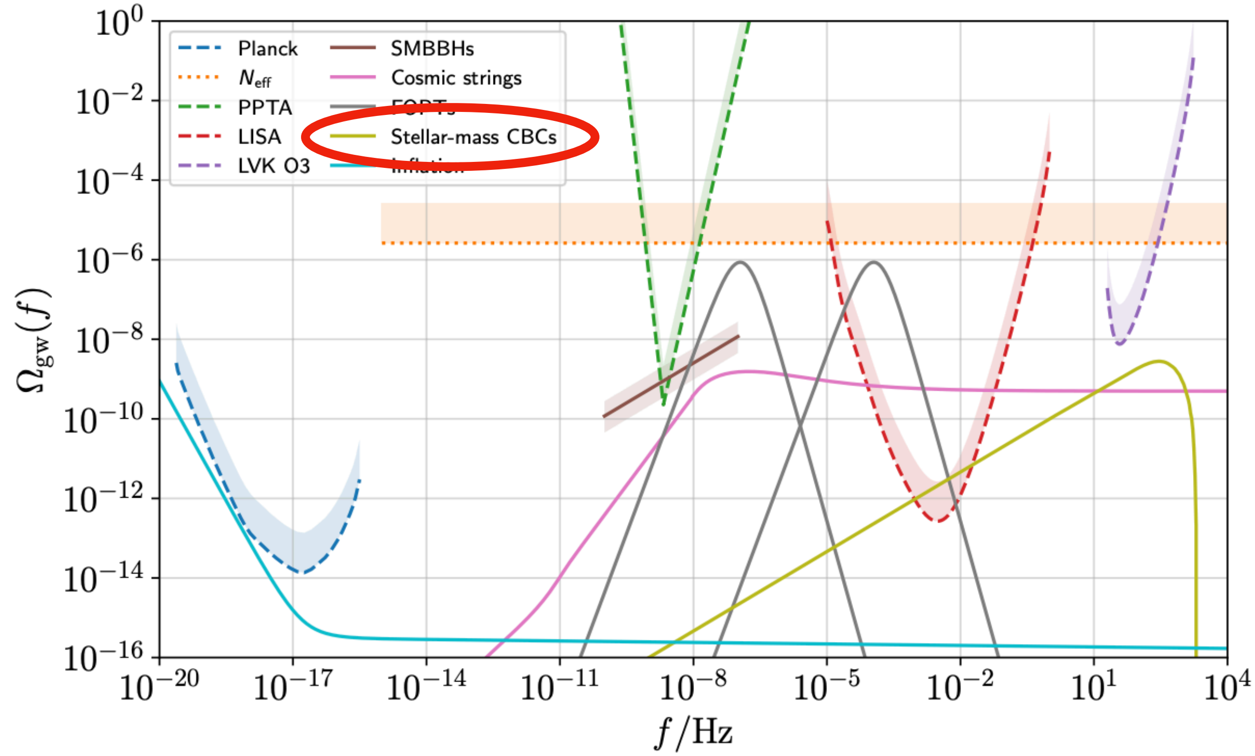
$$\Omega_{\rm GW}(f) = \frac{f}{\rho_c}\int_0^\infty dz \frac{\mathscr{C}(\alpha,\beta,z_p)}{(1+z)H(z)} \frac{R_0(1+z)^\alpha}{1+\left(\frac{1+z}{1+z_p}\right)^\beta} \int d\theta p(\theta) \frac{dE_{\rm GW}(\theta;f_r)}{df_r}$$

This is **horrifying** but it has a nice model solution:

$$\Omega_{\rm GW}(f) = \Omega_{\rm GW}(f_{\rm ref})\left(\frac{f}{f_{\rm ref}}\right)^\alpha$$

which is a **power law!**

GWB Sources: Astrophysical Background



Renzini, et. al. (2022)

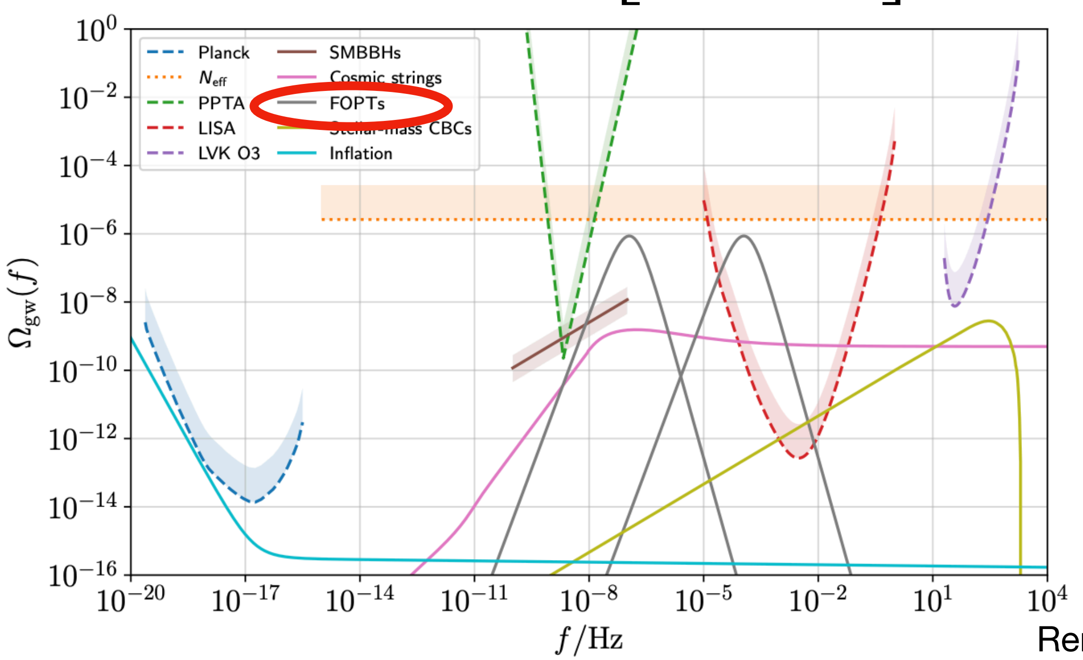
GWB Sources: First Order Phase Transitions

- phase transitions in early universe
- formation of lower-temperature bubbles of different phases of matter or fields
- electroweak and QCD phase transitions are primary contributors
- three main mechanisms:
    - bubble collisions -> shock waves in surrounding plasma
    - sound waves from bubble expansion (greatest GW contributor)
    - turbulence in plasma from bubble motion and growth

GWB Sources: First Order Phase Transitions

Broken power law with smooth turnover:

$$\Omega_{\rm FOPT} = \Omega_* \left(\frac{f}{f_*}\right)^{\alpha_1} \left[1 + \left(\frac{f}{f_*}\right)^{\Delta}\right]^{(\alpha_2-\alpha_1)/\Delta}$$



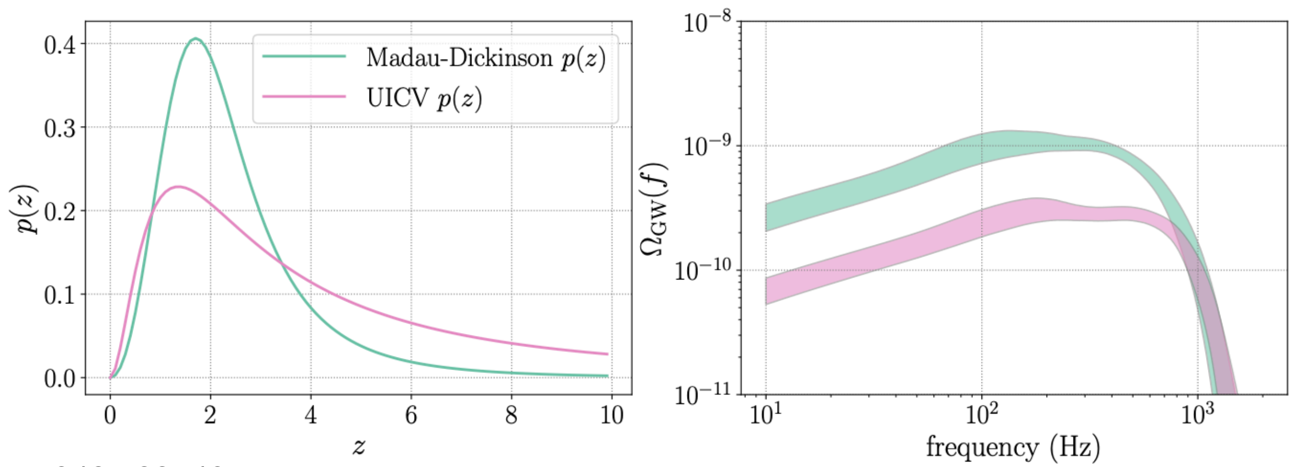
Renzini, et. al. (2022)

LIGO + Current Work

- need multiple detectors with multiple data streams
- cross-correlate many chunks of data: $\hat{C}_{ij}(f;t) = \frac{2}{T} \frac{\mathrm{Re}\left[\tilde{s}_i^*(f;t)\tilde{s}_j(f;t)\right]}{\Gamma_{ij}(f)S_0(f)}$
- signal variance: $\sigma_{ij}^2(f;t) = \frac{1}{2\Delta f T} \frac{P_i(f;t)P_j(f;t)}{\Gamma_{ij}(f)^2 S_0(f)^2}$
  - s_i - Fourier-domain data in i-th detector
  - $\Gamma_{i,j}$ - overlap reduction function
  - P_i - power spectral density for i-th detector (encodes sensitivity)
  - T - observing time
  - S_0 - normalization

LIGO + Current Work

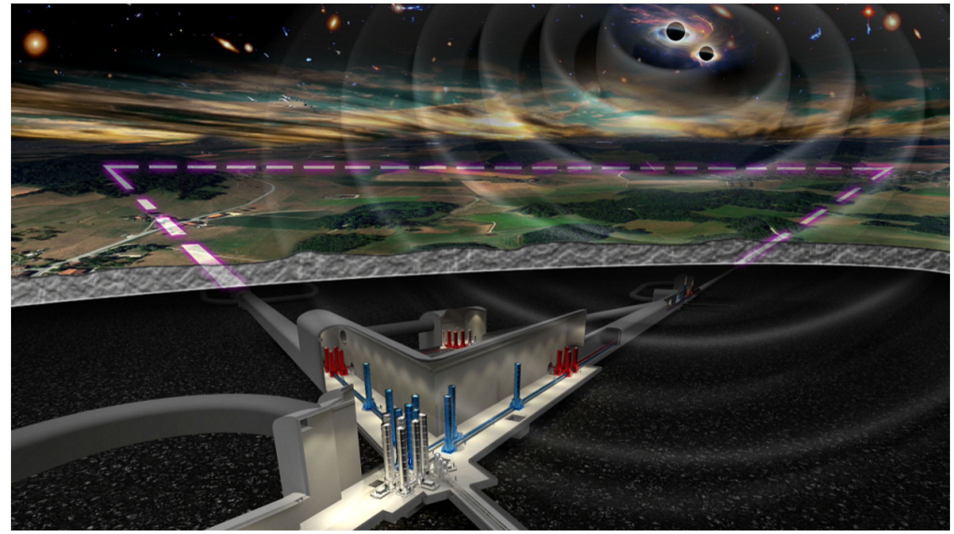
- more continuous observing time reduces variance
- marginalize background over possible population configurations (2407.03742)
    - Bayesian statistics!
    - stochastic sampling with Markov Chain Monte Carlo simulations
    - input physics-informed priors, get posteriors on parameters/distributions



2407.03742

Einstein Telescope (ET)

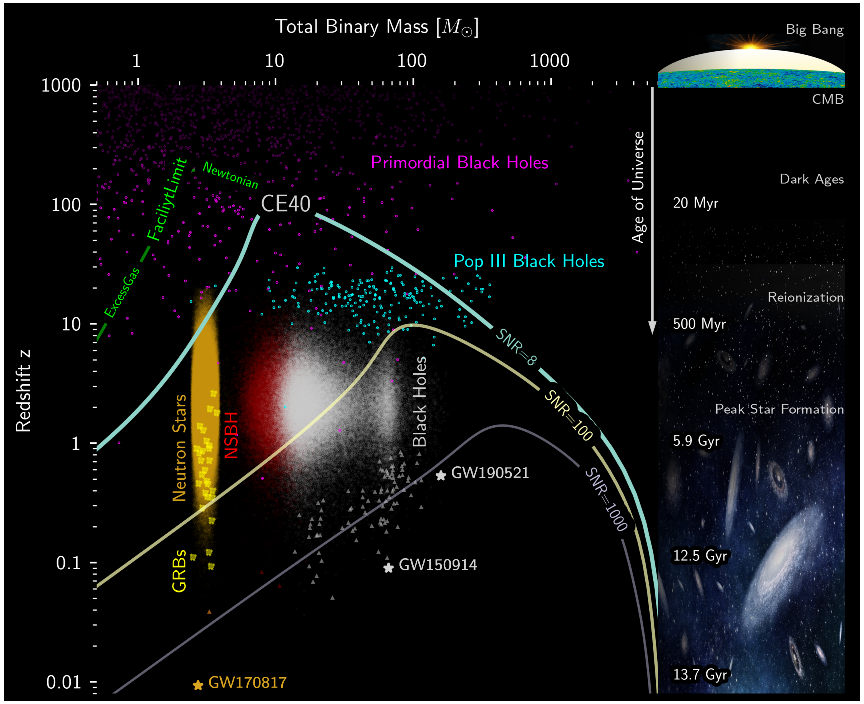
- LIGO - 4km arms
- Virgo - 3km arms
- ET - 10km arms underground
  - kilohertz sensitivity band
- expect to see
  - more stellar and intermediate mass BHs
  - BNS inspiral phase
  - as always, want to test GR
- likely going to live on border of Belgium/Netherlands/Germany



https://www.et-gw.eu/

Cosmic Explorer (CE)

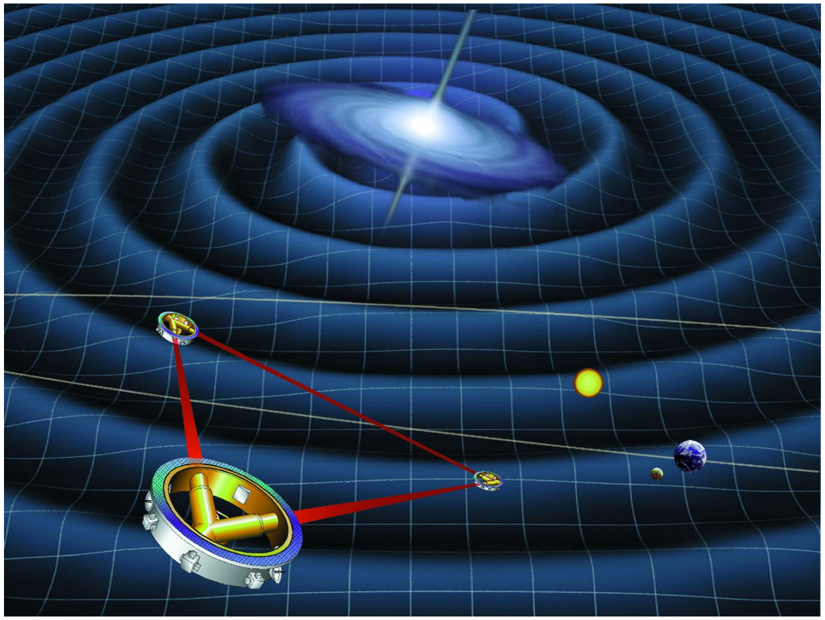
- another new L-shaped detector: 40km on one arm, 20km on the other
- considered a 3G detector with ET and LISA



https://gravitationalwaves.syracuse.edu/about/

Laser Interferometer Space Antenna

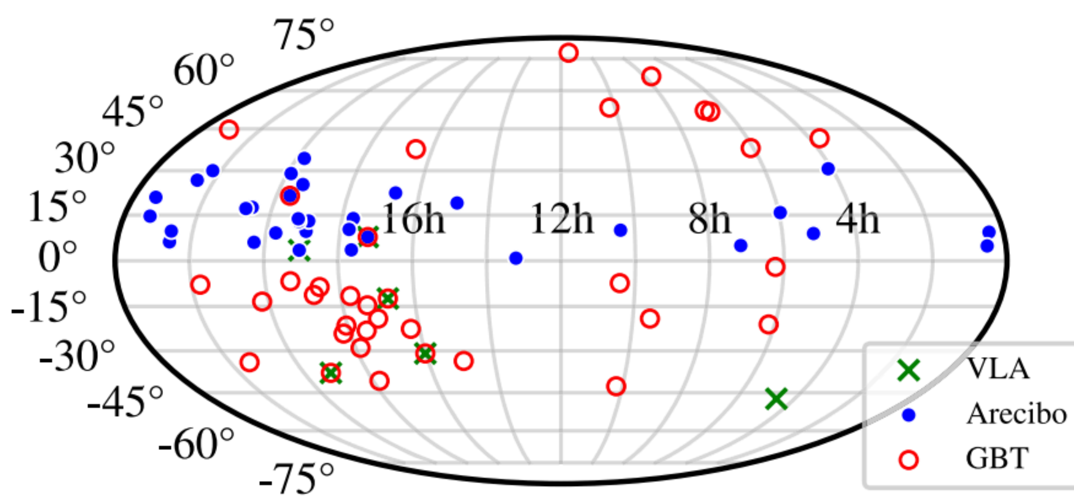
- space-based interferometer
- planned launch ~2035
- ~2.5 million km between each station
- free-floating gold cubes in each station
- successful LISA pathfinder mission paved way
- commissioned for 4 + 6 years of observing
- probe anisotropies to the GWB



https://lisa.jpl.nasa.gov/gallery/lisa-waves.html

Pulsar Timing Array (PTA)

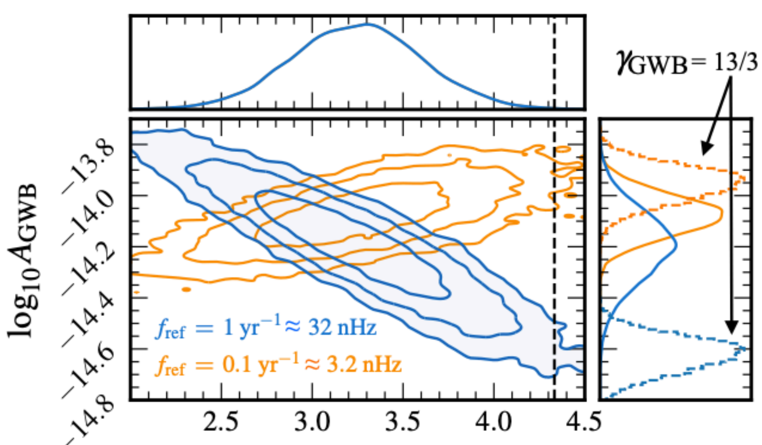
- very precise cosmic clocks
- millisecond pulsars - rotate hundreds of times per second
- analyze their arrival times to look for GWs



arXiv:2306.16213

Pulsar Timing Array (PTA)

- Hellings-Downs Curve
    - residuals of pulsar timing v. angular separation on the sky
    - quasiquadrupolar if GWB exists
    - residuals - change in regular pulsar period due to passing GWs
- 15 yr data set bounded GWB amplitude and spectral index!



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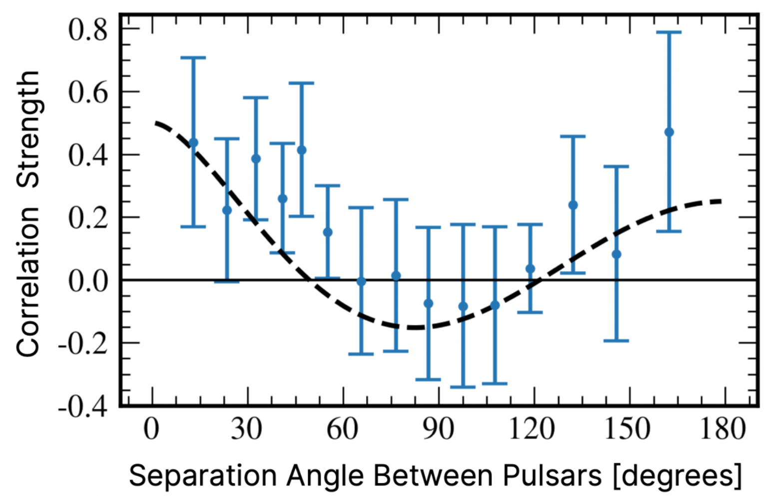

Conclusions

- GWB - superposition of unresolved sources
    - hopefully will resolve many of these with future detectors
- two families of signals that contribute:
    - astrophysical events
    - cosmological/primordial sources
- use Bayesian statistics to bound and understand the GWB right now
- packs in a lot of different astrophysics

- Observation of Gravitational Waves from a Binary Black Hole Merger. [PRL 116, 061102 (2016)]
- Stochastic Gravitational-Wave Backgrounds: Current Detection Efforts and Future Prospects. [arXiv:2202.00178]
- Probing early Universe supercooled phase transitions with gravitational wave data. [Phys. Rev. D 107, 023511]
- Projections of the uncertainty on the compact binary population background using popstock. [2407.03742]
- pygwb: A Python-based Library for Gravitational-wave Background Searches. [2303.15696]
- Cosmic Star Formation History. [arXiv:1403.0007]
- Population Properties of Compact Objects from the Second LIGO-Virgo Gravitational-Wave Transient Catalog. [arXiv:2010.14533]
- https://www.aei.mpg.de/ptas
- The NANOGrav 15-year Data Set: Evidence for a Gravitational-Wave Background. [arXiv:2306.16213]
- Understanding the gravitational-wave Hellings and Downs curve for pulsar timing arrays in terms of sound and electromagnetic waves. [arXiv:1412.1142]
- https://www.et-gw.eu/
- https://cosmicexplorer.org/index.html#overview
- https://www.esa.int/Science_Exploration/Space_Science/LISA_factsheet
- Probing anisotropies of the Stochastic Gravitational Wave Background with LISA. [2201.08782]
- https://nanograv.org/15yr/Summary/Background
- Sampling the full hierarchical population posterior distribution in gravitational-wave astronomy. [arXiv:2502.12156]