SIMULATING eXTREME SPACETIMES

Black holes, neutron stars, and beyond...

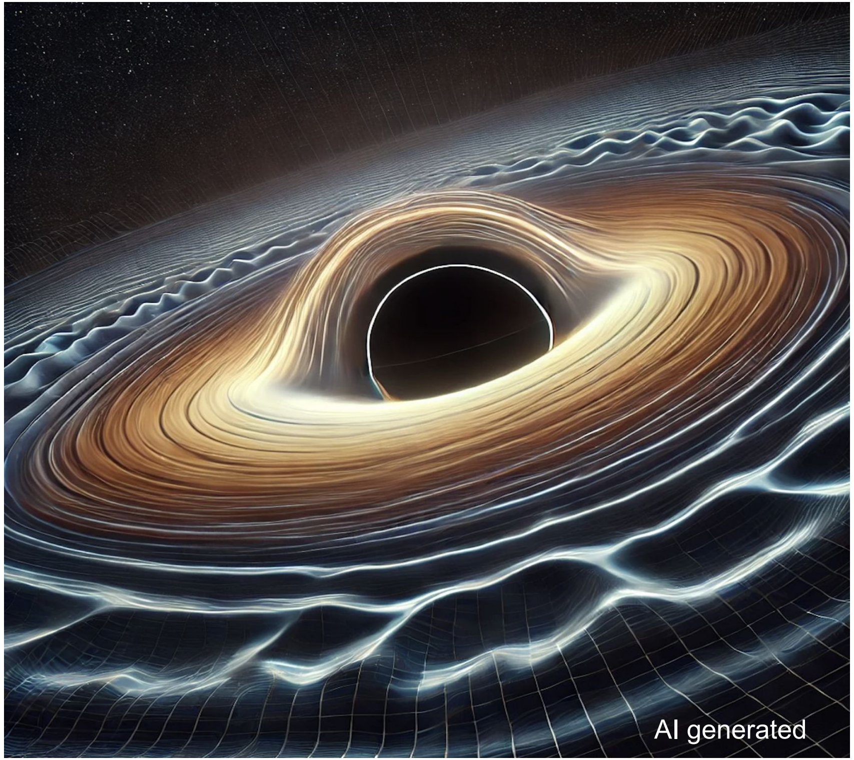
Caltech

RINGDOWN IN IN BINARY BLACK HOLE SIMULATIONS

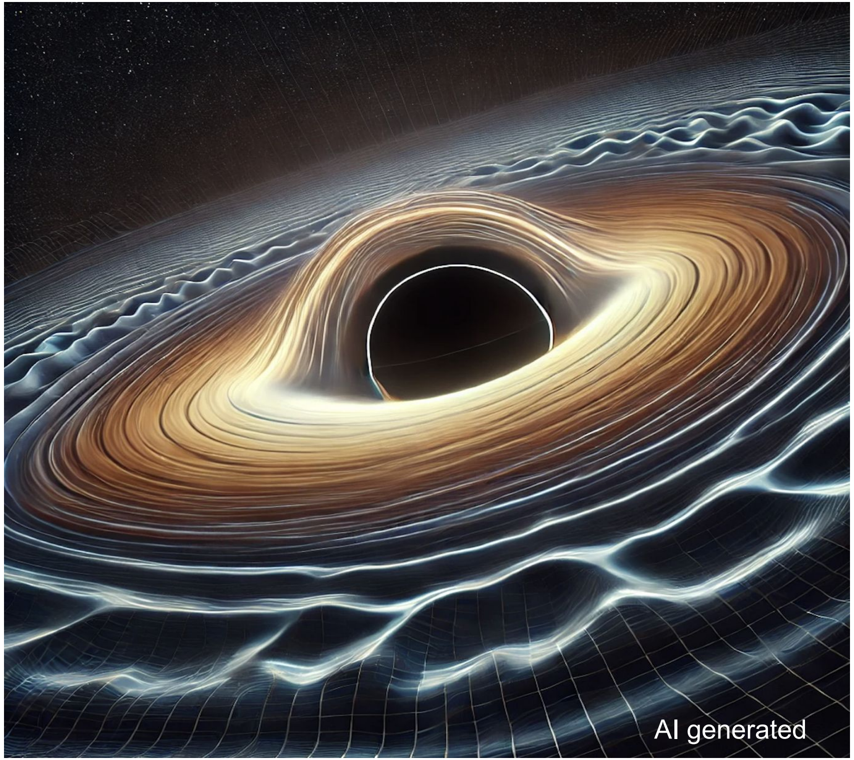
Isabella Pretto

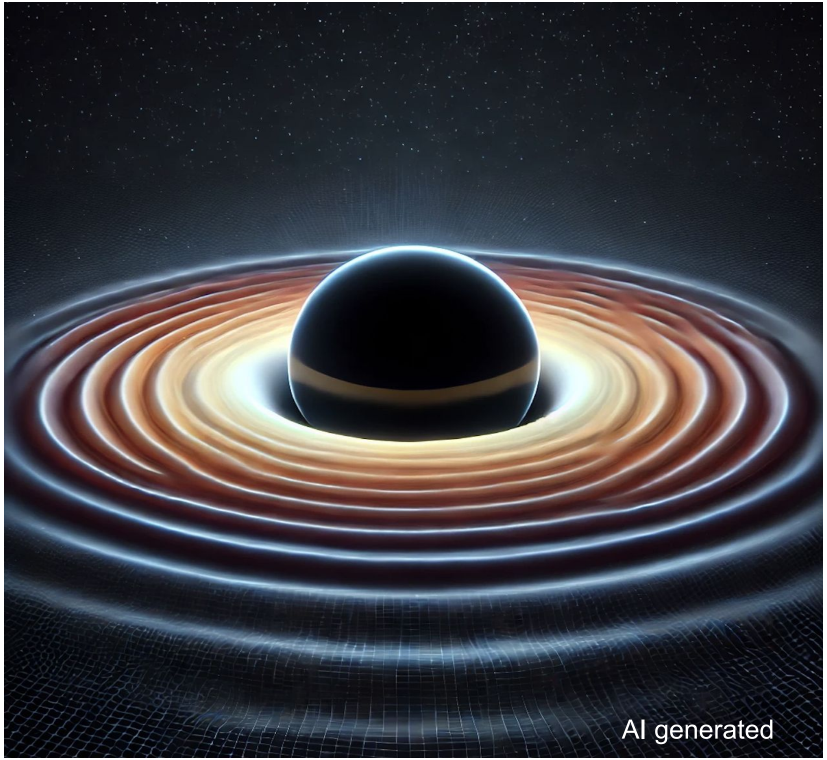
ARC Seminar 03/07/2025

Ringdown



Ringdown





Ringdown

- 1957: Birth of BH perturbation theory (Regge and Wheeler)
- 1963: Kerr solution is discovered
- 1967: Wheeler coins the term ‘black hole’
- 1970-71: Schwarzschild BHs (Regge, Wheeler, Zerilli, Vishveshwara, Press, Davis…)
- 1973: Teukolsky separates the eqs. in Kerr
- 1975: Characteristic frequencies (Chandrasekhar, Detweiler…)

Ringdown - Kerr

- Kerr metric: most general vacuum BH sol. in 4-D asymp. flat
- Curvature invariants → perturbation equations decouple and separate
- Master perturbation eq. for fields of general spin
- Newman-Penrose formalism

$$\Psi_0 = -C_{1313} = -C_{\mu\nu\lambda\sigma} l^\mu m^\nu l^\lambda m^\sigma \,,$$

$$\Psi_4 = -C_{2424} = -C_{\mu\nu\lambda\sigma} n^\mu m^{*\nu} n^\lambda m^{*\sigma} \,.$$

Ringdown - Kerr

- For a spin-s field,

$$\psi(t, r, \theta, \phi) = \frac{1}{2\pi} \int \mathrm{e}^{-\mathrm{i}\omega t} \sum_{l=|s|}^{\infty} \sum_{m=-l}^{l} \mathrm{e}^{\mathrm{i}m\phi} \, {}_sS_{lm}(\theta) R_{lm}(r) \, \mathrm{d}\omega,$$

Ringdown - Kerr

$$\left[\frac{\partial}{\partial u}(1-u^2)\frac{\partial}{\partial u}\right]{}_sS_{lm} + \left[a^2\omega^2u^2 - 2a\omega s u + s + {}_s A_{lm} - \frac{(m+su)^2}{1-u^2}\right]{}_sS_{lm} = 0,$$

$$\Delta\partial_r^2 R_{lm} + (s+1)(2r-2M)\partial_r R_{lm} + V R_{lm} = 0.$$

$$V = 2\mathrm{i}s\omega r - a^2\omega^2 - {}_s A_{lm} + \frac{1}{\Delta}[(r^2+a^2)^2\omega^2 - 4Mam\omega r + a^2m^2 + \mathrm{i}s(am(2r-2M) - 2M\omega(r^2-a^2))].$$

Ringdown - Kerr

➢ Quasinormal modes (QNMs) and boundary conditions

$$\Psi \sim \mathrm{e}^{-\mathrm{i}\omega(t+r_*)}, \qquad r_* \to -\infty (r \to r_+).$$

$$\Psi \sim \mathrm{e}^{-\mathrm{i}\omega(t-r_*)}, \qquad r \to \infty,$$

Ringdown

- After prompt response, quasinormal modes (QNMs)

$$h(u,\theta,\phi) = \sum_{j} \left\{ \substack{\ell\geq 2, |m|\leq \ell, \\ n\geq 0, p=\pm 1} \right\} C_j e^{-i\omega_j t} {}_{-s}S_{(\ell,m)}(a\omega_j)$$

- $\omega_{(l,m,n,p)}$ depends only on mass and spin of final black hole

- Detect at least 2 frequencies— test the no-hair theorem

Ringdown

➢ Second-order perturbation

$$\mathcal{T}\psi = \mathcal{S},$$

$$\omega \equiv \omega_{(\ell_1,m_1,n_1)} + \omega_{(\ell_2,m_2,n_2)}.$$

Fitting procedure

$$h(u,\theta,\phi) \sim \sum_{j} = \left\{ {\ell\geq 2, |m|\leq \ell, \atop n\geq 0, p=\pm 1} \right\} C_j e^{-i\omega_j t}$$

Fitting procedure

- Well-posedness
  – The solution exists
  – It is unique
  – It depends uniquely on the data

- Solving for QNMs is an ill-conditioned problem

- Variable projection algorithm

$$f(\omega, C, t) = c_1\phi(\omega_1, t) + c_2\phi(\omega_2, t) + \ldots + c_n\phi(\omega_n, t)$$

Stability

- ***Measured*** QNM amplitude should decay as ~$e^{t \times \mathrm{Im}[\omega]}$

Stability

- ***Measured*** QNM amplitude should decay as $\sim e^{t \times \mathrm{Im}[\omega]}$
- Fractional variation of C_{QNM} at reference time

$$\boxed{\sigma_{QNM} \stackrel{\mathrm{def}}{=} \sqrt{(\Delta \mathrm{Re}[C_{QNM}]/|C_{QNM}|)^2 + (\Delta \mathrm{Im}[C_{QNM}]/|C_{QNM}|)^2}}$$

Stability

- ***Measured*** QNM amplitude should decay as $\sim e^{t \times \mathrm{Im}[\omega]}$
- Fractional variation of C_{QNM} at reference time

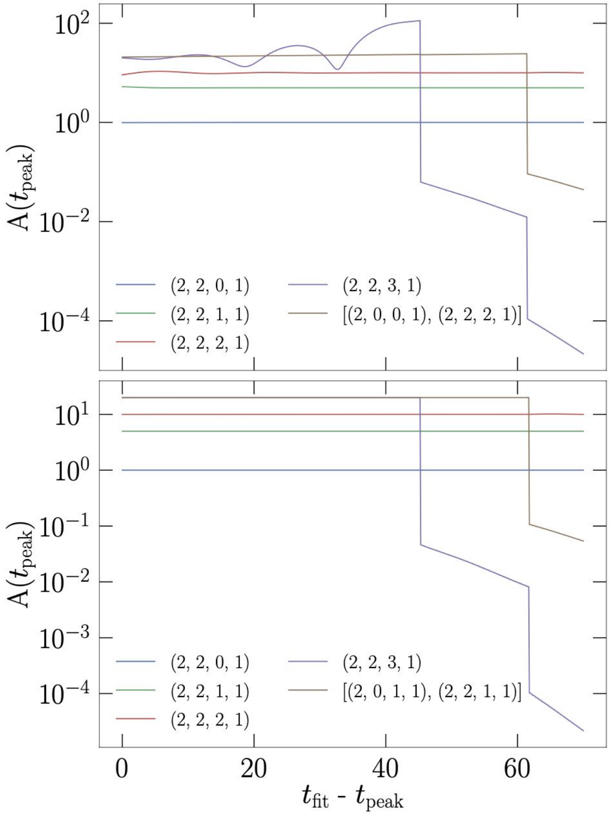
$$\boxed{\sigma_{QNM} \stackrel{\text{def}}{=} \sqrt{(\Delta \mathrm{Re}[C_{QNM}]/|C_{QNM}|)^2 + (\Delta \mathrm{Im}[C_{QNM}]/|C_{QNM}|)^2}}$$

- Require $\boxed{\sigma_{QNM} < \sigma_{max}}$ over $\boxed{\Delta t_{QNM} = -\ln(\Delta A_{min})/\mathrm{Im}[\omega_{QNM}]}$

Stability

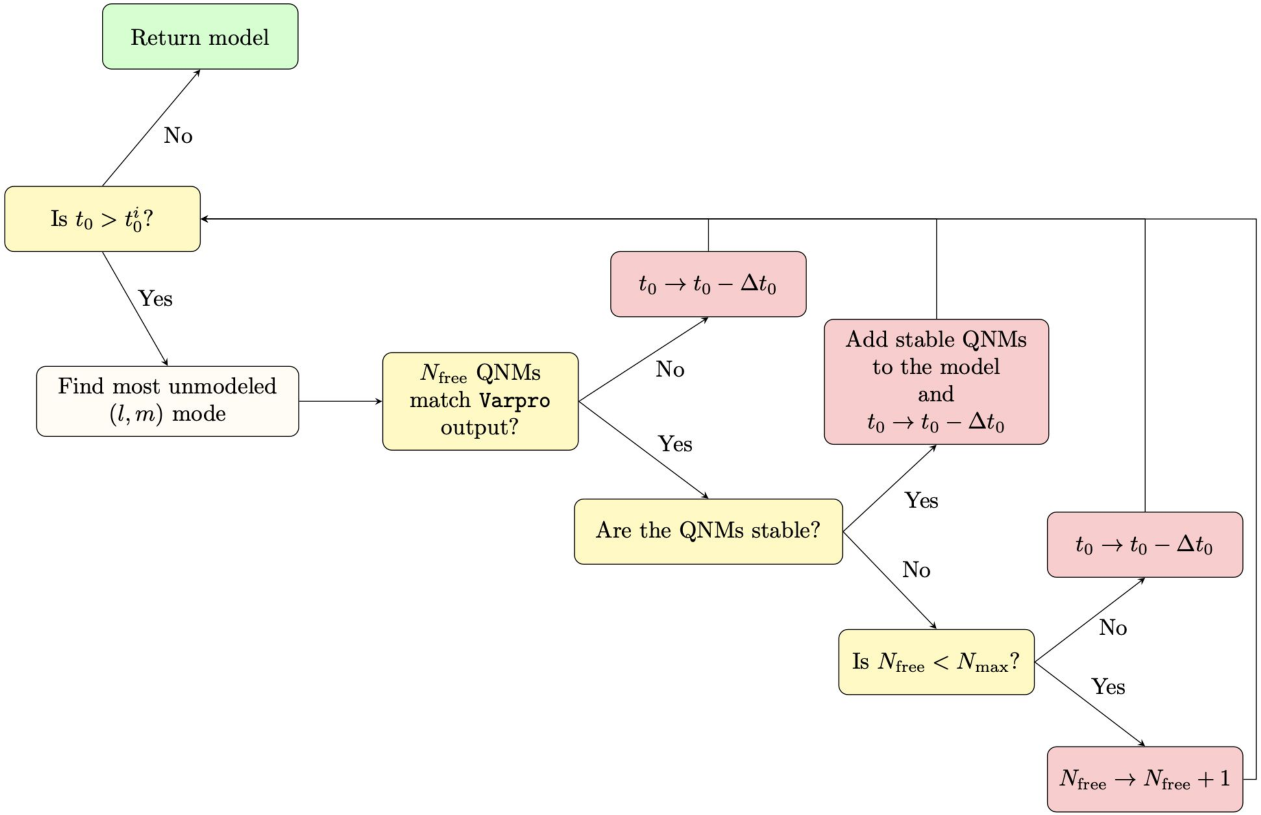
(2,0,1,1)×(2,2,1,1): 0.807 - 0.528i

(2,0,0,1)×(2,2,2,1): 0.801 - 0.530i

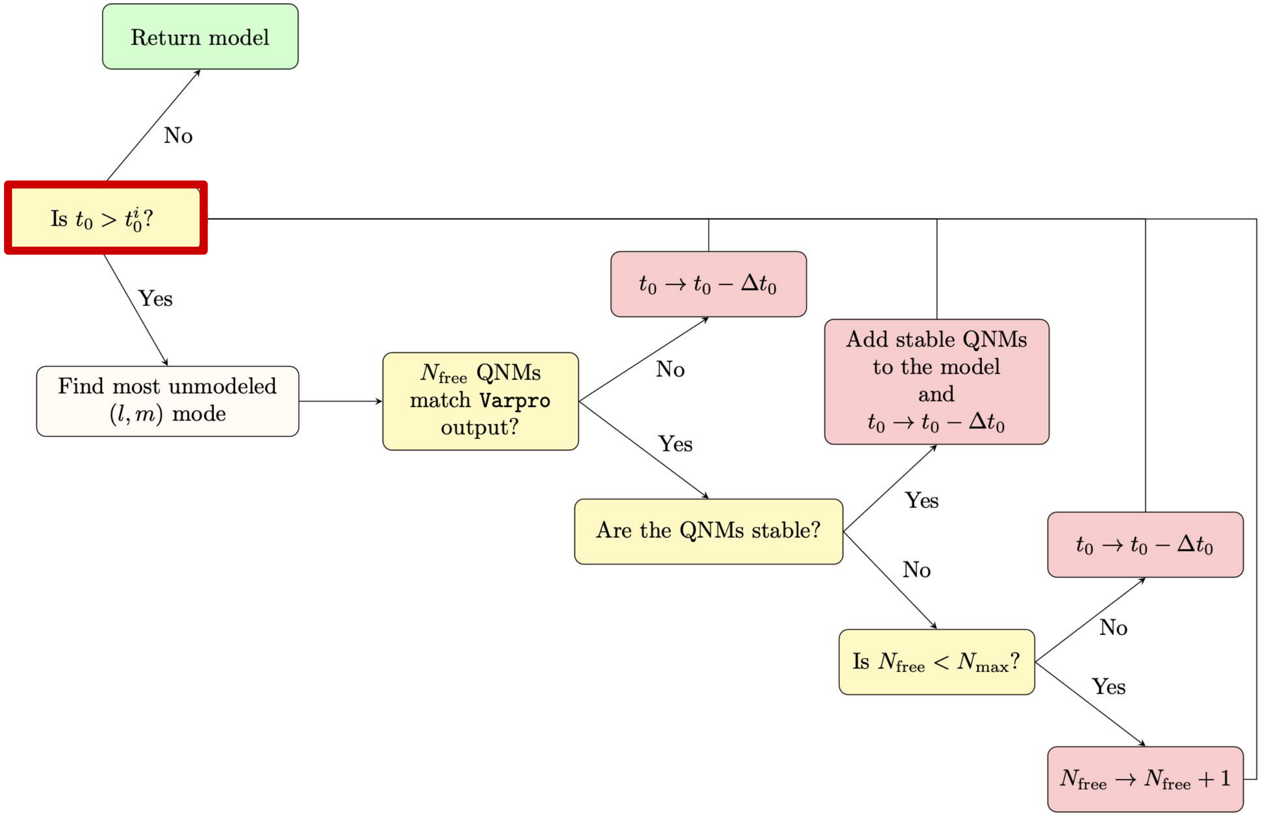


How to automate the search for QNMs in numerical relativity waveforms?

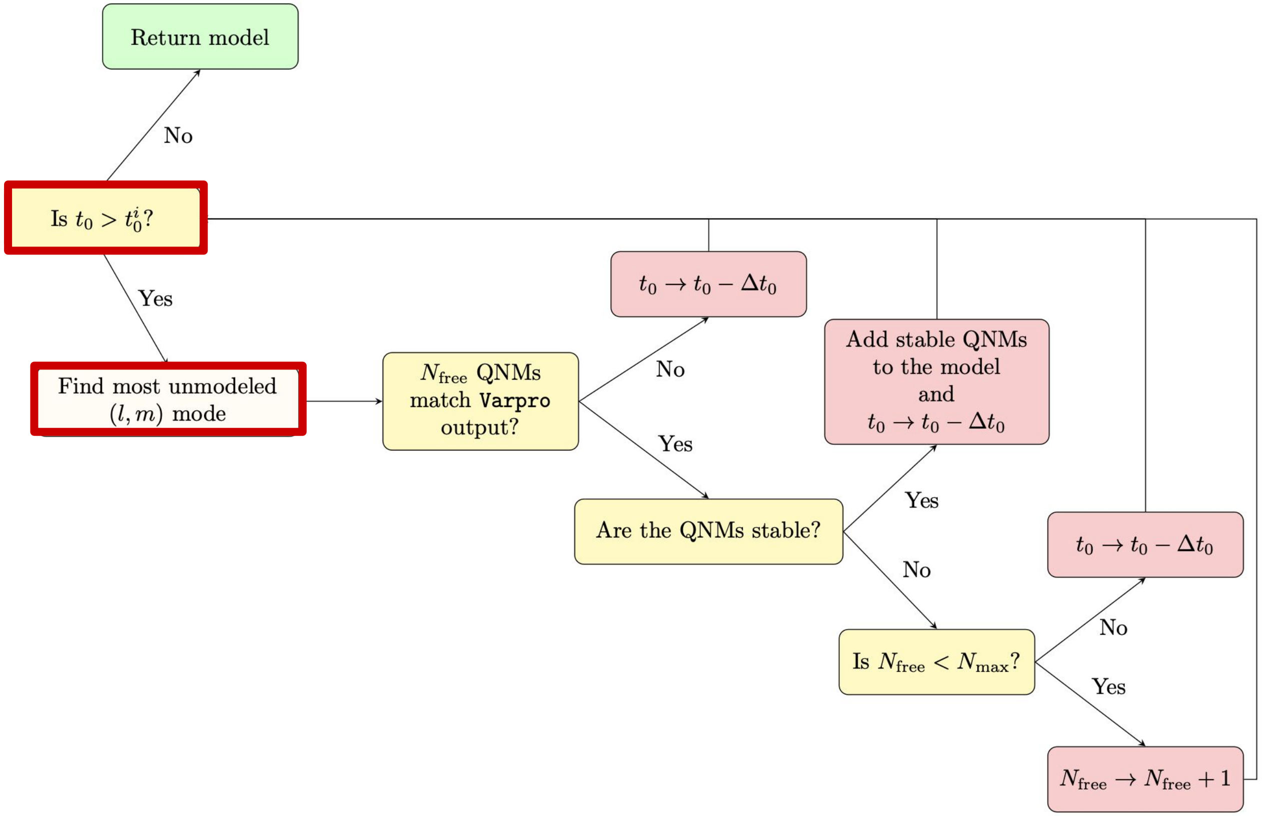
Building a QNM model



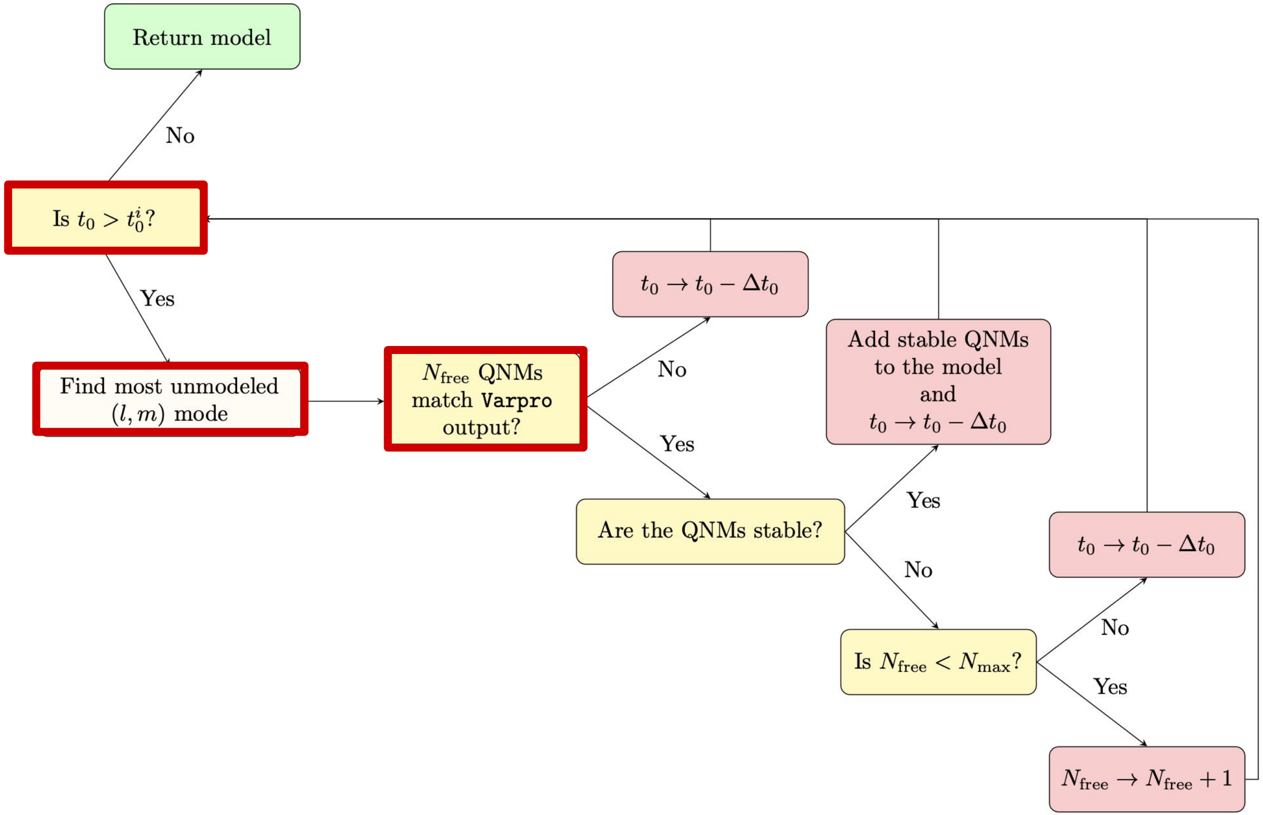
Building a QNM model



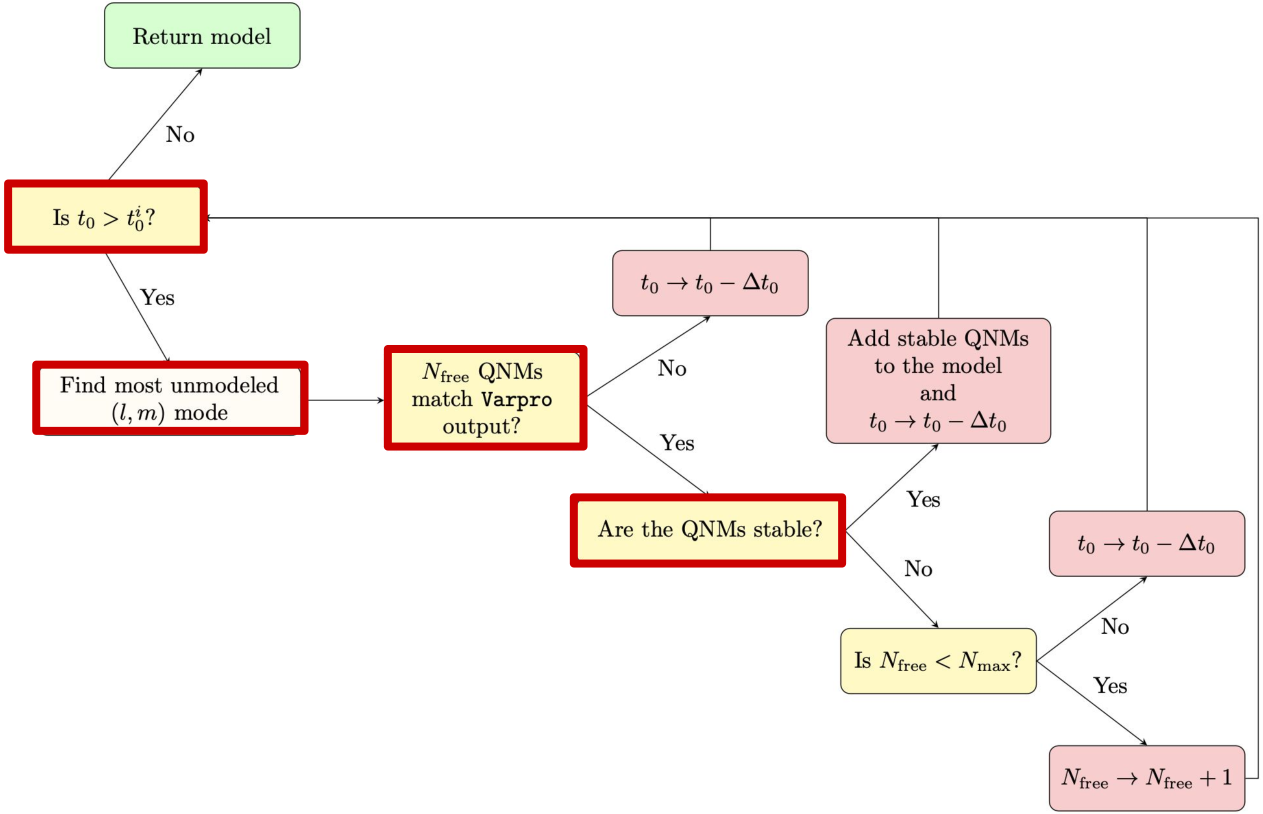
Building a QNM model



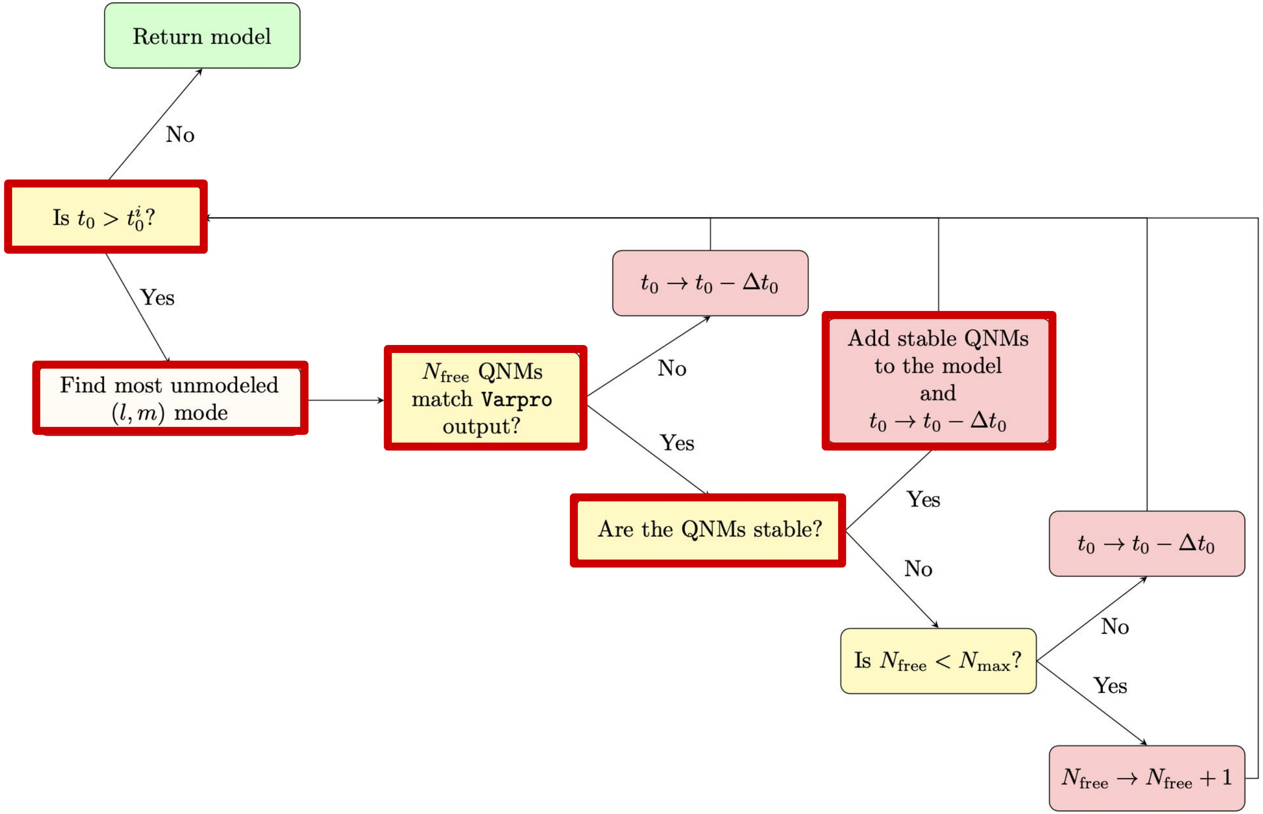
Building a QNM model



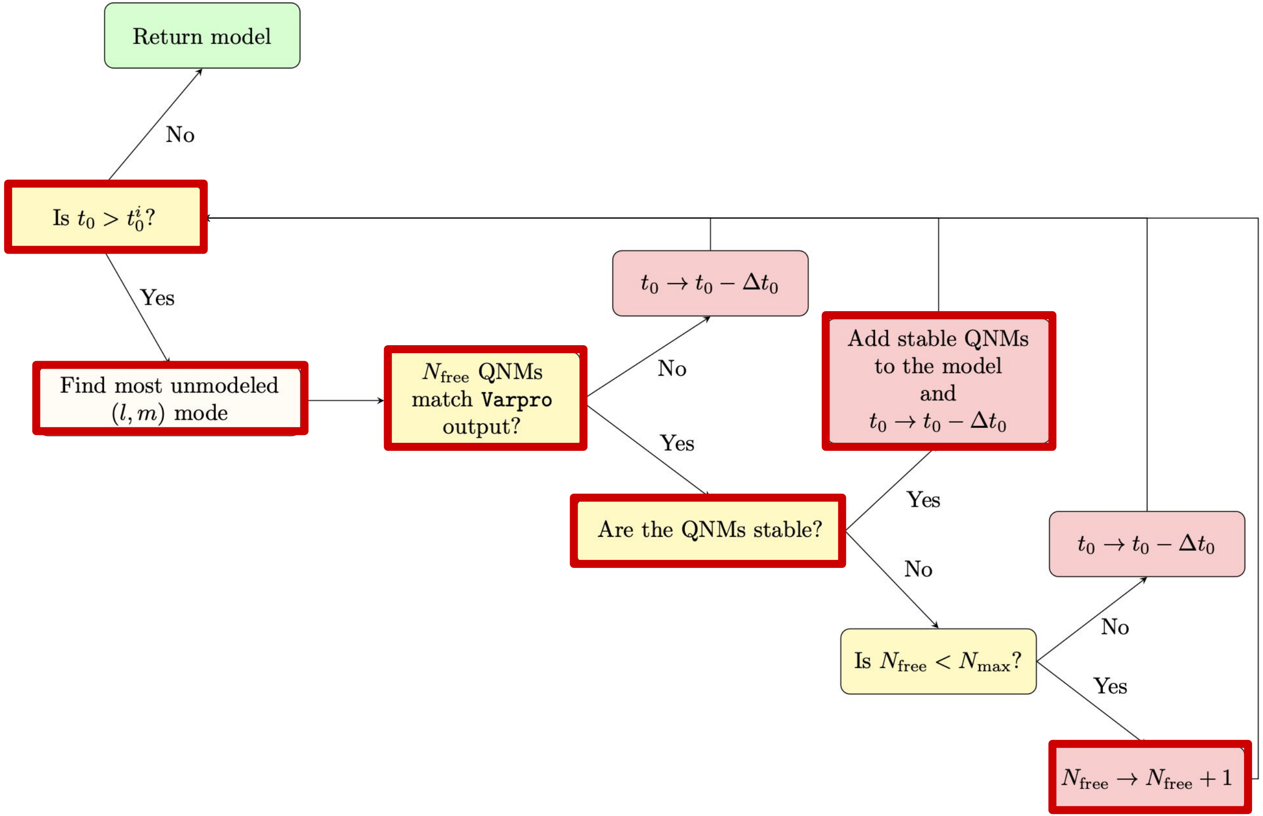
Building a QNM model



Building a QNM model



Building a QNM model



Features

- Iterative fitting

Features

- Iterative fitting

- Fit over multiple harmonics $(2 \leq l \leq 6,\ |m| \leq l)$

Features

- Iterative fitting
- Fit over multiple harmonics $(2 \leq l \leq 6,\ |m| \leq l)$
- Multi-frequency search

Features

- Iterative fitting
- Fit over multiple harmonics $(2 \leq l \leq 6, |m| \leq l)$
- Multi-frequency search
- Stability

Tests

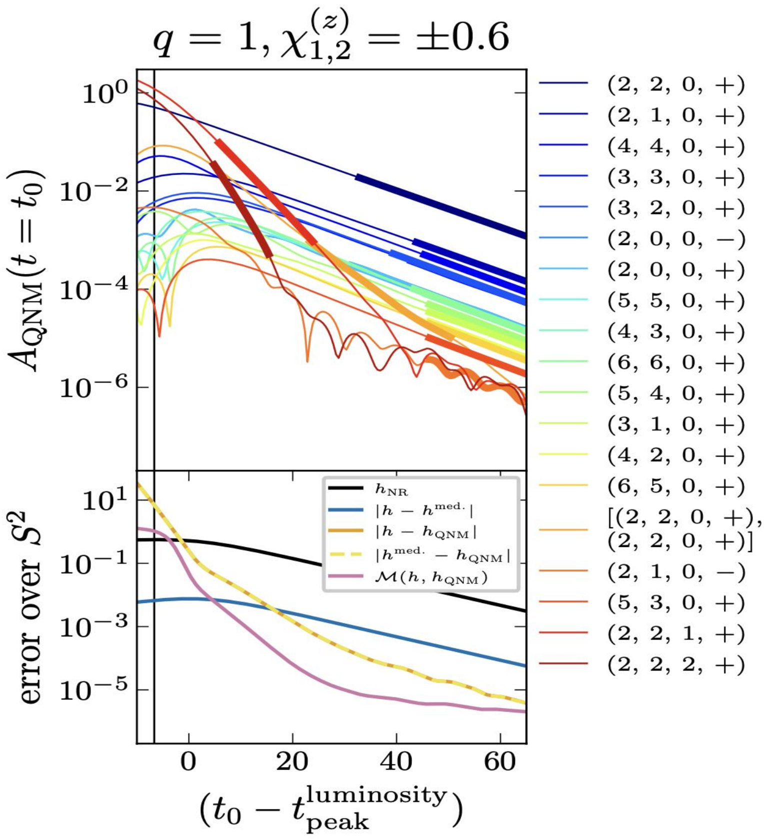
- > 55 mock waveforms, with non-Gaussian/Gaussian noise

$$\sigma_{noise} \in \{10^{-6}, 10^{-5}, \ldots, 0.1\}$$

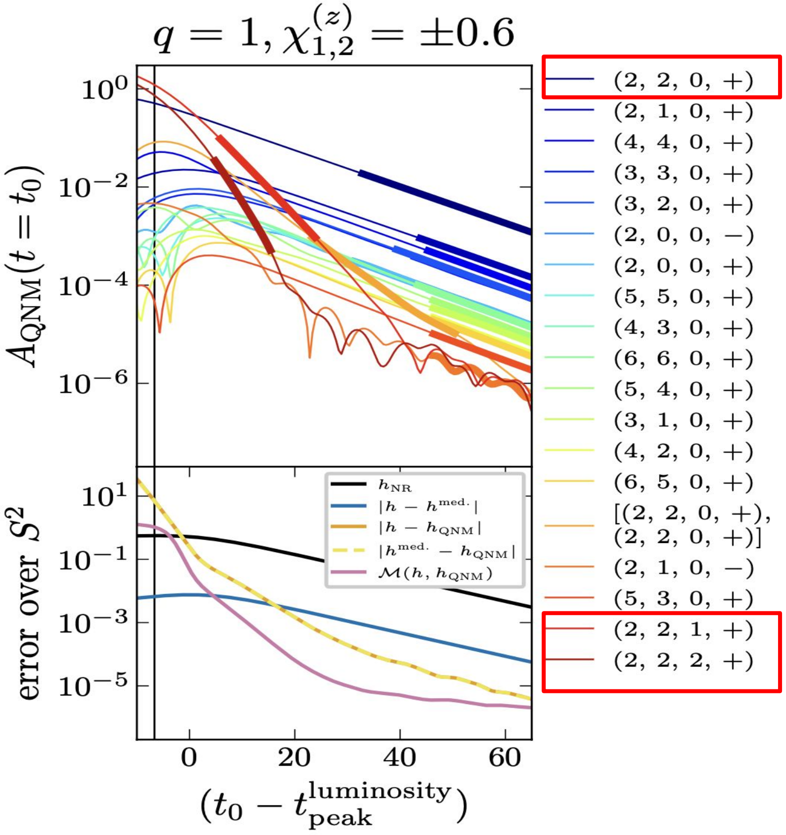
- Find optimal values for A_{min} and σ_{max}

- Validate algorithm

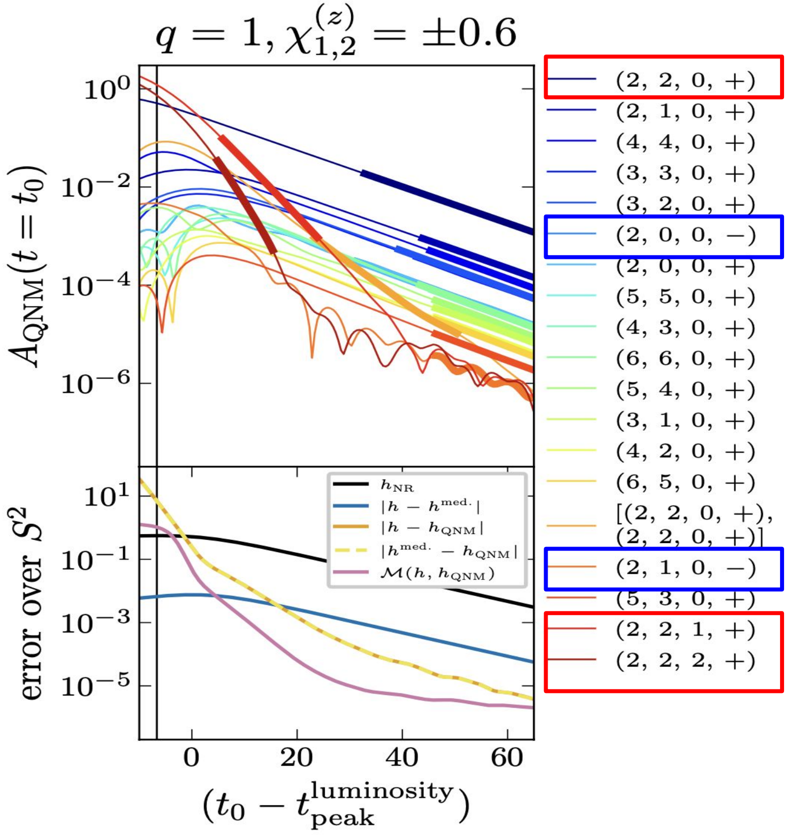
NR waveforms



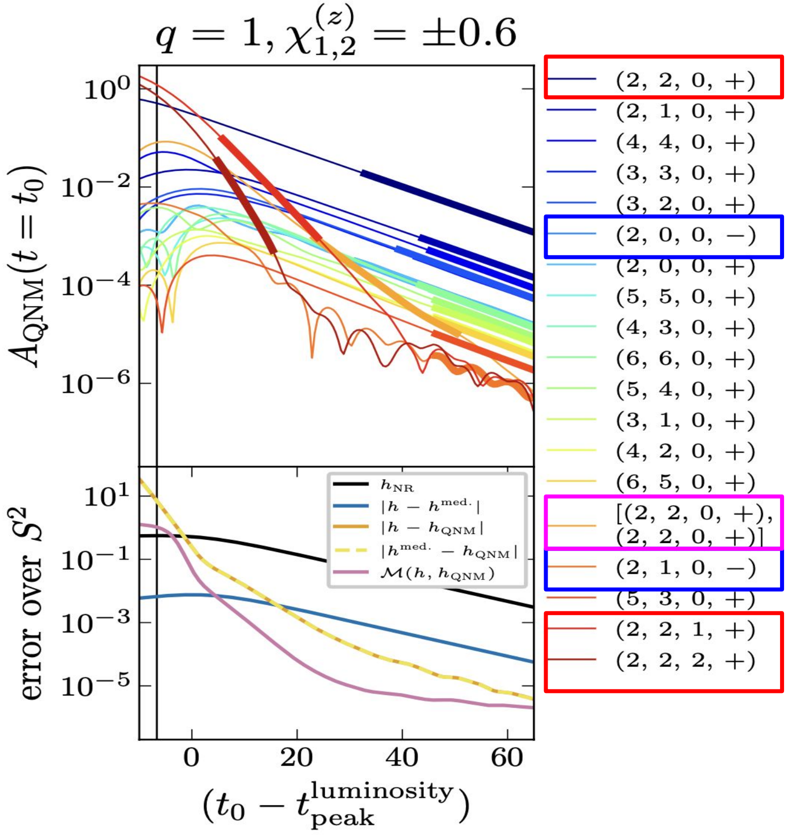
NR waveforms



NR waveforms



NR waveforms



NR waveforms

- Analyze the excitation of QNMs
- ~400 quasi-circular, non-precessing systems with $q < 8$, $|\chi^{(z)}_{1,2}| < 0.8$
- $$C_{\rm QNM} = I_{\rm QNM} E_{\rm QNM},$$

Source function

Excitation factor

NR waveforms

- Analyze the excitation of QNMs
- ~400 quasi-circular, non-precessing systems with $q < 8$, $|\chi^{(z)}_{1,2}| < 0.8$
- $\cancel{C_{\rm QNM} = I_{\rm QNM} E_{\rm QNM},}$ $\qquad E_{\rm QNM} = \frac{1}{I_{(\ell,m)}} e^{-i\omega_{\rm QNM}\Delta t_0^{(\ell,m)}} C_{\rm QNM}$

C_{QNM}

Source function

Excitation factor

Oshita, Cardoso (2024)
arXiv:240702563

NR waveforms

- Source function— flat for *(2, 2, n, +)* overtones?

- Constrain free parameters

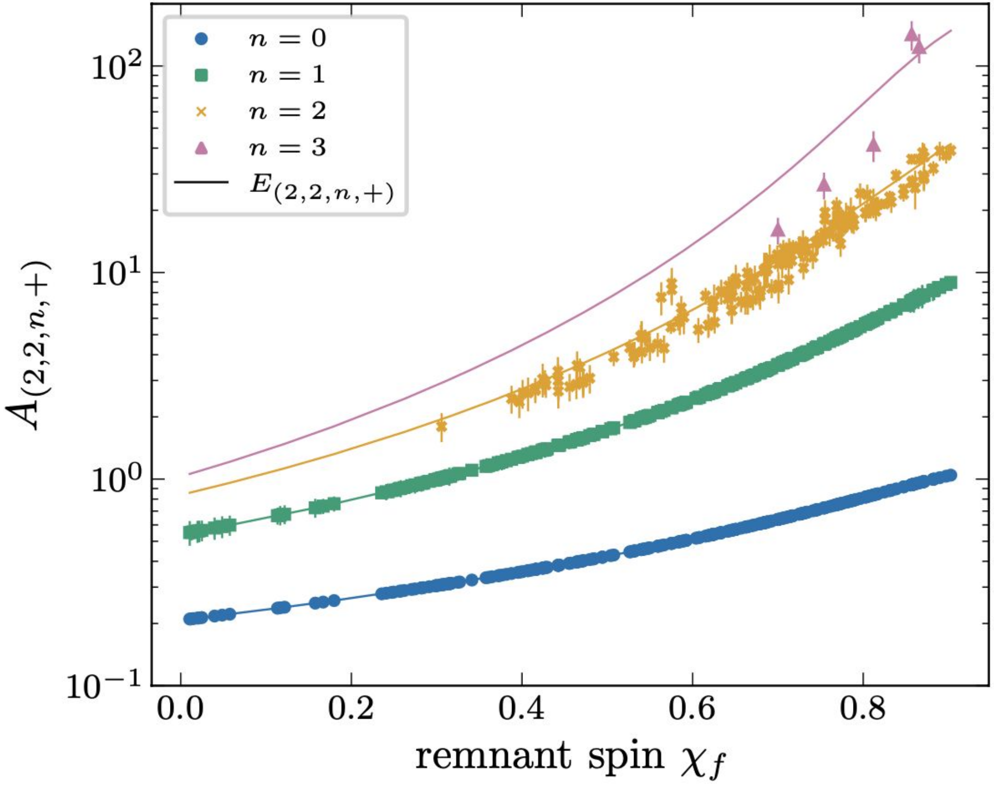
$$\Delta t_0^{(2,2)} = \frac{\ln\left(\left|\frac{E_{(2,2,0,+)}C_{(2,2,1,+)}}{E_{(2,2,1,+)}C_{(2,2,0,+)}}\right|\right)}{\mathrm{Im}[\omega_{(2,2,0,+)} - \omega_{(2,2,1,+)}]}$$

$$I_{(2,2)} = \frac{C_{(2,2,0,+)}}{E_{(2,2,0,+)}} e^{-i\omega_{(2,2,0,+)}\Delta t_0}$$

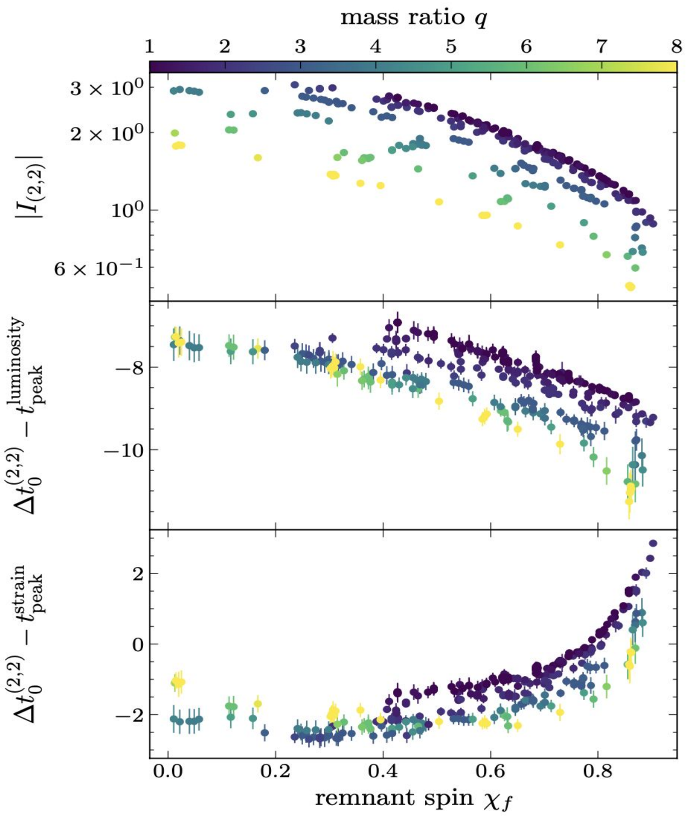
Oshita (2021)
arXiv:2109.09757

Giesler et al. (2024)
arXiv:2411.11269

NR waveforms



NR waveforms



Conclusion

- ➢ Iterative fitting with Varpro + on-the-fly stability analysis
- ➢ Mock and NR waveforms
- ➢ Physics of asymmetric systems and (2,2) excitation
- ➢ Future
  - Different BBH systems
  - Breakdown of constant-QNM model?